Rent Extraction, Political Institutions and the Centralisation of Public Goods Provision

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ABSTRACT. The paper compares decision-making on the centralisation of public goods provision in the presence of regional externalities under representative and direct democratic institutions. A model with two regions, two public goods and regional spillovers is developed in which uncertainty over the true preferences of candidates renders strategic delegation as in Besley and Coate (2003) impossible. Instead, it is shown that the existence of rent extraction by delegates alone suffices to make cooperative centralisation more likely through representative democracy. In the non-cooperative case, the more extensive possibilities for institutional design under representative democracy increase the likelihood of centralisation.

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1. Introduction

In the literature on fiscal federalism, a variety of theoretical reasons for the decentralisation (or symmetrically: against the centralisation) of economic policies are given. The insight that a centralised provision of uniform supply of public goods for a population with heterogeneous preferences in different regions is associated with costs for those whose preferences are not considered can be traced back at least to Oates (1972), and it has subsequently been the starting point for a number of different models. For example, Alesina and Spolaore (1997), in a theory of the optimal size of nations, introduce the fundamental tradeoff between economies of scale in the consumption of public goods and the costs that arise due to heterogeneous preferences under a uniform provision of public goods.

The problem becomes more complicated in the face of regional public goods with spatial externalities. If individuals in \( A \) gain a utility from a public good provided in \( B \) and free-ride on the policies of \( B \), underprovision of the public good is a standard result. Welfare can then be improved through a different channels: If a central authority is politically or technologically restricted to setting a uniform level of public goods in \( A \) and \( B \), then a tradeoff between the welfare gains from internalising spillovers and the welfare losses from a deviation from regional preferences has to analysed. Centralisation may or may not provide a Pareto improvement, depending on which effect is dominant (Alesina et al. 2005).

This tradeoff disappears, however, if the central authority can set regionally different levels of public goods. Prima facie, centralisation that internalises spillovers ought to be
generally Pareto improving under this condition. However, additional caveats have been introduced in models that also take political economy arguments into consideration, such as Besley and Coate (2003) who show that individuals may have an incentive to strategically delegate public goods lovers to the central level, which results in an overprovision of public goods. In a technically similar model, Dur and Roelfsema (2003) show that not only over-, but also underprovision may occur under a centralised regime if the public goods provided have a regional cost component that cannot be shared through the central budget. Similarly, while Persson and Tabellini (1994) argue that centralised regimes are more vulnerable to lobbying and therefore associated with larger budgets, Mazza and van Winden (2002) show that a two-tier government with central and regional authorities is modeled, smaller budgets may result compared to a purely decentralised system.

While the contributions sketched above focus on the effects of (de-)centralisation, a different question concerns the process of institutional change: Under which conditions can we expect centralisation of government to occur? Ellingsen (1998) presents a model where the decision to centralise in the face of spillovers hinges on the size and heterogeneity of regions and shows that different combinations of the relative size of regions and inter- as well as intra-regional heterogeneity are associated with very different probabilities of centralisation. In contrast, a model by Redoano and Scharf (2004) focuses on a comparison of the centralisation decisions under direct and representative democracy. They show that under the latter, a pro-centralization jurisdiction can commit to a reluctant jurisdiction by sending a delegate with preferences close to that of the majority in the reluctant jurisdiction. Therefore, centralisation is more likely to occur under representative democracy in their model.
Regarding the question posed, the model presented here is most closely related to that by Redoano and Scharf (2004): Is centralisation more likely to occur under direct or under indirect democracy? There are, however, important differences in the approach. While the model by Redoano and Scharf understands centralisation as harmonisation, our model allows a centralised regime to set different levels of public good supply over different regions. And while Redoano and Scharf focus on problems of commitment that occur in the process of centralisation, we allow for rent extraction as a general problem of political delegation to appear in the model. Nevertheless, it will be shown that even under these very different assumptions, the result presented by Redoano and Scharf is robust: centralisation is more likely to occur under representative compared to direct democracy.

2. **Set-up of the model**

In this section, a simple model with two regions is introduced where different local public goods are preferred and intrajurisdictional utility-spillovers may occur. Suppose that, partially resembling the specifications of Besley and Coate (2003) as well as Dur and Roelfesma (2005), an individual $I$ in one of the regions $i, j \in 1, 2$ with $i \neq j$ has the utility function

$$U^I_i = x + \lambda_i^I [b(g_i) + \gamma b(g_j)]$$

(1)

where $x$ is the amount of private goods consumed, $g$ is the quantity of a local public good, $0 < \gamma \leq 1$ is a spillover parameter signifying for instance geographical proximity.
between the two regions and $b(\cdot)$ is a strictly concave, increasing valuation function for public goods.

Furthermore, we assume that the parameter $\lambda$ denoting the preferences for public goods is distributed over an interval $(0, \bar{\lambda})$ such that the median preference is identical in both regions, $\lambda^m_i = \lambda^m_j = 1$. Furthermore, let the public good preferences of candidates for political office be distributed over the same interval that, if a representative is drawn randomly, her \textit{ex ante} expected public goods preference is $E(\lambda^r) = 1$. The assumption that the true preferences of a representative are not observable is probably not unrealistic, since we frequently observe in practical politics that announced platforms are tailored strategically to win majorities and are not identical with the actual preferences of candidates running for office. Technically, this assumption allows us to leave problems of strategic delegation out of consideration – this would require that the true preferences are known – and concentrate on the effect that rent extraction has on the decision to centralise.

Both regions are inhibited by an equal number of $n$ individuals. Also, the technologies of public goods provision are identical in both regions and public goods are financed by lump sum taxes such that each individual has to give up one unit of the private good in order to allow the provision of one unit of a local public good. In other words, regions are completely identical as far as the technicalities of public good provision are concerned, with the only qualification that different kinds of public goods are supplied in the two regions. For example, region $i$ might supply a publicly funded theatre that can also be visited by citizens of $j$, while $j$ supplies a public park that is also visited by citizens of $i$. 
Therefore, the main difference between our model and that of Besley and Coate (2002) is that the latter is interested in strategic delegation, while the former emphasises the imperfections of the political process in representative democracies.

Finally, we assume that a representative who is in office can secure a rent from every unit of a local public good that is supplied under his legislation. Thus, while a representative formally has to pay the same head tax as every other citizen, his effective contribution is only $\sigma g_i$ with $0 < \sigma < 1$ (i.e., he secures a rent of $(1 - \sigma)$ per unit of public goods).

3. Three regimes of public good provision

Regarding the political institutions of public goods provision, three different regimes are distinguished.

Decentralised public good provision. In this case, the median voter in each jurisdiction is interested to solve

$$g_i^{Dm} = \arg \max_{g_i > 0} \ U_i^m - g_i$$

(2)

which leads to the first order condition of $\frac{\partial b(g_i)}{\partial g_i} = 1$ for an optimal $g_i^{Dm}$. A representative on the other hand aims at

$$g_i^{Dr} = \arg \max_{g_i > 0} \ U_i^r - \sigma g_i$$

(3)

where $U_i^r = x + \lambda^r [b(g_i) + \gamma b(g_j)]$, which leads to the first order condition of $\frac{\partial b(g_i)}{\partial g_i} = \sigma/\lambda^r$ for an optimal $g_i^{Dr}$. A median voter endowed with perfect knowledge would thus choose a representative with a preference for public goods $\lambda^r = \sigma$. Without credible signalling mechanisms for the representatives’ true public goods preferences, however,
the expected true value of \( \lambda' \) is \( E(\lambda') = 1 \). In this case, representative democracy is associated with expected overspending, the actual extent of which will depend on influences not formally considered here, such as the likelihood of ex post punishment via retrospective voting. If, on the other hand, a budget referendum is obligatory or can be organised at sufficiently low cost, overspending will be avoided.

Centralised, cooperative public good provision. To analyse this institutional framework, we assume, closely related to Weingast (1979), that both elected representatives express their wishes for the level of public goods in their own jurisdiction and engage in pork-barelling thereafter. Also, a simple cost-sharing rule is assumed, which arranges that the total costs of public good provision are devided equally between both jurisdictions. Each representative then solves

\[
g^C_i = \arg \max_{g_i > 0} U^r_i - \frac{\sigma}{2}(g_i + g_j)
\]

so that the first order condition is \( \partial b(g_i)/\partial g_i = \sigma/2 \). From the symmetry assumption, it follows that \( g^C_j = g^C_i \). Letting the median voter in each jurisdiction decide about which public good levels he would prefer under this regime of cost sharing would, on the other hand, lead to the first order conditions \( \partial b(g_i)/\partial g_i = 1/2 \) and \( \partial b(g_j)/\partial g_j = 1/2\gamma \). Measured against the median preferences, a collusive agreement between regional representatives would therefore always lead to overspending, even if \( \sigma = 1 \), as long as spillovers are not complete and \( \gamma < 1 \).

Centralised, non-cooperative public good provision. Suppose that, while the cost-sharing rule from the cooperative regime remains the same, the spending levels are not decided upon by collusive agreement between representatives, but that decision-making power is
delegated to centralized institutions of collective decision-making. Suppose further that, on the central level, a decision is made between a spending proposal drafted in $i$ and a spending proposal drafted in $j$. Then, $p$ denotes the probability that a proposal from $i$ is chosen at the central level, and correspondingly, $(1 - p)$ is the probability of choice for the proposal from $j$. The uncertainty about the outcome of the centralized decision will usually have multiple causes: voter turnout may be different across jurisdictions, in a representative system constituencies may be shaped to influence the result in a certain direction and so on.

If a non-cooperative spending proposal is passed by a referendum, the median voter on the local level will choose

$$\{g_i^{Nm}, g_j^{Nm}\} = \arg \max_{g_i > 0, g_j > 0} U_i^{Nm} - \frac{1}{2}(g_i + g_j)$$

with the first order conditions being $\frac{\partial b(g_i)}{\partial g_i} = \frac{1}{2}$ and $\frac{\partial b(g_j)}{\partial g_i} = \frac{1}{2}$. If, on the other hand, a non-cooperative spending proposal is drafted by a representative, his choice will be

$$\{g_i^{Nr}, g_j^{Nr}\} = \arg \max_{g_i > 0, g_j > 0} U_i^{Nr} - \frac{\sigma}{2}(g_i + g_j),$$

yielding as first order conditions $\frac{\partial b(g_i)}{\partial g_i} = \sigma / 2$ and $\frac{\partial b(g_j)}{\partial g_j} = \sigma / 2$.

4. Pathways to a centralisation of spending competencies

*Centralization via referendum.* Presuming that the status quo is a decentralised setting and that we are interested in processes of centralisation, the interesting question is to see under which conditions the electorate or representatives are inclined to agree to a
centralisation of public spending. Comparing the median voter’s utility under a decentralised, direct-democratic regime with that under a cooperatively centralised regime, it is easy to see that centralisation will be preferred if

\[ b(g^c_i) + \gamma b(g^c_j) - \frac{1}{2}(g^c_i + g^c_j) > b(g^{Dm}_i) + \gamma b(g^{Dm}_j) - g^{Dm}_i. \]  

(7)

Since it follows from our symmetry assumption that \( g^c_i = g^c_j \) and \( g^{Dm}_i = g^{Dm}_j \), we can note

**Lemma 1.** If \( \sigma \) is sufficiently large to ensure that the left hand side of (8) is not smaller than 1/2, then there exists always a \( \gamma^*_1 \leq 1 \) so that

\[ \frac{b(g^c_i) - b(g^{Dm}_i)}{g^c_i - g^{Dm}_i} > \frac{1}{1 + \gamma} > 1 \]

(8)

and the median voter prefers a centralised over a decentralised provision of public goods.

**Proof.** Eq. (8) follows immediately from (7). From our first order conditions, it follows that the slope of \( b(\cdot) \) at \( g^{Dm}_i \) equals unity, while it equals \( \sigma/2 \leq 1/2 \), at \( g^c_i \). The left hand side of (8) displays the slope of the secant that runs through \( g^c_i \) and \( g^{Dm}_i \). Therefore, and due to the concavity of \( b(\cdot) \), the value of the left hand side has to be strictly smaller than unity and larger than \( \sigma/2 \). For very small values of \( \sigma \), the slope of the secant may be smaller than the right hand side of (8) even for \( \gamma = 1 \). Thus, centralization will only be favoured if the rents appropriated by the representatives are sufficiently small and the spillovers are sufficiently large. \( \Box \)
If the decentralised, direct-democratic regime competes against a non-cooperative, centralised regime with direct-democratic decision-making over the spending proposals, centralisation will be preferred if

\[ p[b(g_i^{Nm}) + \gamma b(g_j^{Nm})] + (1 - p)[b(g_j^{Nm}) + \gamma b(g_i^{Nm})] - \frac{1}{2}(g_i^{Nm} + g_j^{Nm}) > b(g_i^{Dm}) + \gamma b(g_j^{Dm}) - g_j^{Dm}. \]  

Note that the costs are not state-dependent due to the symmetry assumption; the same amount will be spent on public goods regardless of which spending proposal is implemented, but it will be differently allocated across regions. Based upon this inequality, we can state

**Lemma 2.** For any \( p \in [0, 1] \), there exists a \( \gamma^*_2 \leq 1 \) that is sufficiently large to make centralisation the preferred regime of public goods provision. For \( p = 1 \), centralisation is preferred for any \( \gamma \in [0, 1] \).

**Proof.** See the appendix.

If \( v(\gamma) \) denotes the expected benefits from centralisation and \( w(\gamma) \) denotes the expected extra costs, then, as is shown in the proof of Lemma 2, \( w(1) \) and \( v(1) \) are independent of \( p \), whereas the curve of \( v(\gamma < 1) \) rotates to the southeast with declining \( p \) and may even become negative for a combination of low values of \( p \) and \( \gamma \). In other words, the interval \( [\gamma^*_2, 1] \) where centralisation is preferred shrinks with a declining \( p \). The obvious problem with centralisation decisions is that not both jurisdictions can have \( p \approx 1 \) at the same time. If it is very likely that the proposal from \( i \) succeeds on the central level, then it has to be very unlikely that the proposal from \( j \) succeeds. From these considerations follows
Lemma 3. The interval $[\gamma^*_2, 1]$ where both median voters favour centralisation is the largest, when $p = 1/2$.

Proof. It is obvious that the interval of consensual centralisation is the largest, when both median voters have the same threshold spillover level for favouring centralisation. Given our symmetry assumptions, this is the case at $p = 1/2$. $\square$

Centralisation by consenting representatives. Decision-making on the centralisation of spending competencies is highly path-dependent. If budgetary decisions in the local jurisdictions are subject to a popular referendum, it is usually not possible for representatives to decide upon the centralisation of spending decisions – the centralisation decision itself would have to be legitimised via a referendum. Thus, the status quo for centralisation by consenting representatives are local jurisdictions with representative decision-making – in other words, we assume that representatives can not on their own authority suspend local direct democracy by creating a centralised representative system. For a representative to favour cooperative centralisation, it is then necessary that

$$\begin{aligned} b(g^c_i) + \gamma b(g^c_j) - \frac{\sigma}{2} (g^c_i + g^c_j) > b(g^{Dr}_i) + \gamma b(g^{Dr}_j) - \sigma g^{Dr}_i \end{aligned}$$

(10)

Solving this inequality leads to

Lemma 4. There exists a level of rent extraction $\bar{\sigma} \in [0, 1)$ for which cooperative centralisation will be preferred by representatives for a level of spillovers $\gamma^*_3 < \gamma^*_4$. Contrary to direct democracy, decision-making by representatives also ensures that even with very high levels of rent extraction, there is a spillover-level for which centralisation
is preferred.

*Proof.* See the appendix.

If, on the other hand, non-cooperative centralisation is to be attained, $\sigma$ is a rather unreliable instrument to increase the range of spillovers for which centralisation is preferred. The reason is simple: Cooperative centralisation via referendum involves a delegation of decision-making powers to representatives on the central level; non-cooperative centralisation involves no such thing, because in this case, centralised decision-making remains subject to a popular referendum. Even with a low $\sigma$, the threat of excessive spending under a centralised regime does not exist, because a referendum is necessary. Due to the missing necessity of delegation, voters are less reluctant to centralise in the non-cooperative case. If, for instance, $b(g) = a \cdot g^\alpha$ with $a > 0, 0 < \alpha < 1$ is chosen as the specification for the valuation function, then $\sigma$ has no impact at all on the value of $\gamma^*$. For other specifications, such as $b(g) = ln(1 + a \cdot g)$, the effect of even very high levels of rent extraction is diminutively small.

Nevertheless, even with $\sigma$ not playing a role, centralisation is more likely to occur under a representative regime if under direct democratic centralisation $p \neq 1/2$. It follows from *Lemma 3* that the range of spillovers for which centralisation is commonly preferred in both jurisdictions will be maximised if $p = 1/2$. There are, though, many reasons that may lead to unequal winning probabilities for the two spending proposals: there may be differences in the culture of political participation, the costs of getting to the urn may be higher in a more rural compared to a more urban jurisdiction and so on. In a direct democracy, where a majority of the entire electorate decides, it is hardly feasible to shape
formal political institutions in order to manipulate $p$. Under representative democracy, on the other hand, instruments to manipulate $p$ are available such as the purposeful shaping of constituencies. If this is possible, then under representative democracy the range $[γ^*, 1]$ where centralisation is favoured can be extended finding formal political institutions for the central level that ensure that $p$ converges towards $1/2$. These considerations, along with Lemmas 1-4, lead to

**Proposition 1.** Representatives are more inclined to favour centralisation of spending competencies than voters in direct-democratic decision-making, since

(i) under cooperative central decision-making, the prospect of additional rent-extraction makes the centralised solution relatively more alluring to representatives than to citizens and

(ii) under non-cooperative central decision-making, a representative system allows for the adjustment of $p$ via the choice of appropriate formal institutions in the case that $p \neq 1/2$ at the outset.

5. **Conclusions**

The main result of the paper is that in a political environment with uncertainty regarding the true preferences of candidates for political office, where strategic delegation is not feasible, the mere existence of rents that can be extracted from holding political office suffices to make centralisation more unlikely in a direct-democratic framework compared
to a representative democracy if decision-making on the central level is made cooperatively. If, on the other hand, central decision-making is non-cooperative and the decisive voter is for some reason more likely to come from one region than from the other, then under a one-man-one-vote principle only a representative system allows (for example through gerrymandering) to move $p$ on the central level closer to $1/2$ and thereby increase the willingness to centralise.

Obviously, the representatives’ tendency to centralise may be mitigated by influences not formally considered here, such as the threat of punishment through retrospective voting. But since it is well known that direct democracy leads to tighter control of politicians compared to representative democracy, such mitigating influences to not principally threaten our result: Representative democracy often enough offers the necessary niches to centralise against the will of the median voter, e.g. by centralising at the beginning of a term and hoping for prospective or myopic voting in the next elections, or by accompanying an unpopular centralising decision with a popular decision elsewhere.

6. References


**Appendix**

Proof of Lemma 2. In a first step, we will show that for \( p = 1 \), centralisation is preferred for any level of spillovers. If \( \gamma = 1 \), the first order conditions from section 2 always lead to \( g_{i}^{Nm} = g_{j}^{Nm} \). The symmetry assumption ensures that \( g_{i}^{D} = g_{j}^{D} \). Then, (9) collapses to

\[
2 [b(g_{i}^{Nm}) - b(g_{i}^{D})] > g_{i}^{Nm} - g_{i}^{D}
\]  \( \text{(11)} \)

\[
\Rightarrow \frac{b(g_{i}^{Nm}) - b(g_{i}^{D})}{g_{i}^{Nm} - g_{i}^{D}} > \frac{1}{2}
\]  \( \text{(12)} \)
which is always true, since at \( g_i^{Nm} \), \( \partial b(g)/\partial g = 1/2 \), and the slope of the secant necessarily assumes a higher value than that. If \( \gamma = 0 \), (9) collapses to

\[
b(g_i^{Nm}) - b(g_i^D) > \frac{g_i^{Nm}}{2} - g_i^D
\]  

Adding \( g_i^{Nm}/2 \) to each side and sorting leads to

\[
\frac{b(g_i^{Nm}) - b(g_i^D)}{g_i^{Nm} - g_i^D} > 1 - \frac{g_i^{Nm}}{2(g_i^{Nm} - g_i^D)}.
\]  

Since \( g_i^{Nm} > g_i^D \), the right hand side can be rewritten as

\[
1 - \frac{z g_i^D}{2g_i^{Nm}(z - 1)} \rightarrow 1 - \frac{z}{2(z - 1)} \quad \text{with} \quad z > 1
\]  

For any \( z \in (1, \infty] \), the right hand side never assumes a value higher than 1/2. The inequality is always true and for \( \gamma = 0 \), centralisation will always be preferred if \( p = 1 \).

Concerning other values of \( \gamma \), there is a complication as far as the benefits of centralisation are not necessarily rising monotonously with \( \gamma \). Let

\[
v = b(g_i^{Nm}) - b(g_i^Dm) + \gamma [b(g_j^{Nm}) - b(g_j^Dm)]
\]  

denote the expected benefits and

\[
w = \frac{1}{2} (g_i^{Nm} + g_j^{Nm}) - g_i^Dm
\]  

denote the expected additional costs from centralisation. Then we have

\[
\frac{\partial w}{\partial \gamma} = \frac{1}{2} \frac{\partial g_j^{Nm}}{\partial \gamma}
\]  

which, given the first order conditions, is necessarily positive. On the other hand,

\[
\frac{\partial v}{\partial \gamma} = b(g_j^{Nm}) - b(g_j^Dm) + \frac{\partial b(g_j^{Nm})}{\partial g_j^{Nm}} \frac{\partial g_j^{Nm}(\gamma)}{\partial \gamma}
\]  

which, after inserting the first order condition, can be written as

\[
\frac{\partial v}{\partial \gamma} = b(g_j^{Nm}) - b(g_j^Dm) + \frac{1}{2} \frac{\partial g_j^{Nm}(\gamma)}{\partial \gamma}
\]
Because the difference between the first two terms will be negative for small \( \gamma \) and because, as can be inferred from the first order conditions, \( g_j^{Nm}(\gamma) \) is either convex with a relatively flat slope for small values of \( \gamma \), or linear, \( v \) may be declining in an interval \((0, \tilde{\gamma}]\) and rises monotonously thereafter.

With \( v(\gamma) \) being convex, \( w(\gamma) \) rising strictly monotonously and \( v(0) > w(0) \) as well as \( v(1) > w(1) \), it is a necessary condition for \( v(\gamma) < w(\gamma) \) at any \( \gamma \in (0, 1) \) that \( v(\tilde{\gamma}) < w(\tilde{\gamma}) \) with \( \tilde{\gamma} \) being exactly that value of \( \gamma \), where the slopes of \( w \) and \( v \) are identical. Equating both partial derivatives yields the condition that \( b(g_j^{Nm}) = b(g_j^{Dm}) \), which is the case exactly at \( \tilde{\gamma} = 1/2 \). Equating \( v(\tilde{\gamma}) \) and \( w(\tilde{\gamma}) \) and keeping in mind that in this case, \( g_j^{Nm} = g_{ij}^{Dm} \), we find that \( v(\tilde{\gamma}) > w(\tilde{\gamma}) \) if

\[
\frac{b(g_j^{Nm}) - b(g_j^{Dm})}{g_j^{Nm} - g_j^{Dm}} > \frac{1}{2},
\]

which is always the case, since once again the left hand side is the slope of the secant and can, due to our first order conditions, not be smaller than \( 1/2 \). Therefore, for \( p = 1 \), centralisation is preferred irrespective of the degree of spillovers.

The next step is to show that even for \( p = 0 \), it is possible that centralisation is preferred. For this purpose, it is sufficient to look at (9), where it is obvious that with \( \gamma = 1 \), the left hand side of the inequality assumes the same value at \( p = 0 \) and at \( p = 1 \). Therefore, the argument that has been made for \( p = 1, \gamma = 1 \) is also valid for \( p = 0, \gamma = 1 \). \( \Box \)

**Proof of Lemma 4.** Taking into consideration that, due the symmetry assumptions, \( g_i^C = g_j^C \) and \( g_i^{Dr} = g_j^{Dr} \), (10) can be written as

\[
\frac{(b(g_i^C) - b(g_i^{Dr}))}{g_i^C - g_i^{Dr}} > \frac{\sigma}{1 + \gamma}.
\]
Let $\alpha(\sigma)$ denote the slope of the secant between $g_i^C$ and $g_i^{Dm}$ in the direct democracy case (i.e., the left hand side of (8), and $\beta(\sigma)$ denote the slope of the secant between $g_i^C$ and $g_i^{Dr}$ in the case of representative democracy. Then, under direct democracy, centralisation will be preferred for any

$$\gamma_1^* > \frac{1}{\alpha(\sigma)} - 1 \quad \text{with} \quad 1 > \alpha > \frac{\sigma}{2}$$

whereas in a representative democracy,

$$\gamma_3^* > \frac{\sigma}{\beta(\sigma)} - 1 \quad \text{with} \quad \sigma > \beta > \frac{\sigma}{2}$$

warrants centralisation. With $\sigma = 1$, the first order conditions for the public goods levels are identical and so are the threshold spillover levels, $\gamma_3^* = \gamma_1^*$. Due to the strict concavity of $b(\cdot)$ and the first order conditions for the optimal spending levels, it is necessarily true that for all $\sigma \in [0, 1), \alpha(\sigma) > \beta(\sigma)$. For the threshold spillover levels from where centralisation is preferred, we know that $\gamma_1^* > \gamma_3^*$ holds if

$$\frac{1}{\alpha(\sigma)} - 1 > \frac{\sigma}{\beta(\sigma)} - 1$$

$$\Rightarrow \beta(\sigma) > \sigma \alpha(\sigma).$$

From this it follows that if $\sigma$ is sufficiently small, $\sigma < \bar{\sigma}$ with $\bar{\sigma} = \beta(\sigma)/\alpha(\sigma)$, then $\gamma_3^* < \gamma_1^*$ holds. Since both $\alpha(\sigma)$ and $\beta(\sigma)$ converge to the same limit with $\sigma \to 0$, there has to exist some $\sigma > 0$ for which the above inequality holds.

It is easily checked that for some specifications of $b(g)$, such as $b(g) = a \cdot g^\alpha$ with $a > 0, 0 < \alpha < 1, \bar{\sigma} = 1$ holds, because in this case, $\sigma \alpha(\sigma)$ is convex with $\alpha(1) = \beta(1)$ and $\lim_{\sigma \to 0} \sigma \alpha(\sigma) = 0$ and $\beta(\sigma)$ is concave or linear with $\lim_{\sigma \to 0} \beta(\sigma) = 0$. \□