Districts, Party Discipline, and Polarization

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Extremely preliminary and incomplete.
Please do not circulate.

Abstract

What is the value of a party, beyond the value of its candidates? We propose a model in which parties act as a "brand name": they help identify candidates who are not too distant from some "national platform". Voters benefits from this, because they learn whether they prefer such party candidates or the independent candidate. Hence, for the party, sticking to a national platform may be costly. We show that parties compensate this by polarizing more when a) they are more powerful; b) districts are sufficiently heterogeneous. Interestingly, parties can be powerful in two polar cases: when they impose a tight party line, or when they are very flexible.

1 Introduction

Why would voters prefer candidates that belong in a party, rather than independent candidates who are free from party ties? That is, what is the value of a party, beyond the value of its candidates? We propose a model in which parties act as a "brand name": they manage to select candidates that are not too distant from an endogenously chosen "national platform". Some voters thus benefit from electing a candidate a party, since they have more information ex-ante about the policies that will be implemented ex-post. Yet, sticking to a national party line also has a drawback: party politicians are less free to adapt their policies to local needs.

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Expressed more concisely, the stricter is party discipline, the more information is elicited by the party tag, but the less can candidates adapt to local policy preferences.

We consider a country composed of a set of districts, each of which is identified by its median voter. There is one election. The elected candidate will implement his preferred policy each time he has agenda setting power. Hence, the utility of the median voter is directly related to the policy preferred by the elected candidate.

To have a chance of winning the seat attached to a given district, parties must find a *local* candidate. Candidate selection is random: each party proposes a randomly chosen citizen to be their candidate. We assume that if the citizen is sufficiently close to the national party line, he accepts the offer ("how close" depends on party discipline – see below). If he turns down the offer, the party makes the offer to another citizen, and so on until a candidate is found or until the pool of potential candidates is exhausted.

Further, we assume that each district also has a local party, which is independent from national parties. By assumption, any citizen in the district is willing to run under the local party flag. Hence, his possible ideologies are distributed according to same distribution as ideologies in the district. In *expected* terms, this implies that his policy perfectly adapts to local preferences.

By contrast, the candidate of a national party is bound by the party "brand name". We represent the strictness of the party discipline by the size of the set of tolerated policies. The smaller is the set, the less freely can candidates adapt their policy to their preference. For this reason, by assumption, they simply refuse to join the party if their preferred policy lies outside the tolerated set. The outcome of this selection procedure is that party members will appear to promote very similar policies, the stricter is party discipline.

Clearly, in districts that are much more "left-wing" than the party, this implies that the candidate of the party will appear relatively "right-wing" by local standards. Conversely, in "right-wing" districts, the candidate of the party will appear relatively "left-wing" by local standards.

Our first result is that districts that are too distant from the platform put forth by the national parties will prefer regional parties. Expressed differently, in first-past-the-post elections, it is not *individuals' heterogeneity* that induces the entry of "niche" parties, but *interregional* heterogeneity.

Second, we show that the value of a party is highest in two polar cases: when party discipline is very strong, and when it is very lax. Intermediate levels of party discipline make national parties the weakest compared to local parties (independents).

Third, the equilibrium polarization of party policies depends on both district heterogeneity and party strength. In particular, the more heterogeneity and the stronger the parties, the more polarized are equilibrium platforms

Fourth, if districts are rather homogeneous and national parties have little internal discipline, then there will be an overlap between the set of ideologies implemented by each party: the right-most candidate in the left party is more right-wing than the left-most candidate in the right-wing party. Instead, if districts are highly heterogenous, if national parties are weak, or if discipline is very strong, then there will be no overlap between the ideologies of the two parties. That is, the right-most candidate of the left party is more left-wing then the left-most candidate of the right-wing party.

Last, we find that parties benefit from maintaining a very tight discipline on their members if the country is characterized by rather homogeneous districts, and very lax party discipline if the country is composed of highly heterogeneous districts.

Review of the literature

`[TBW]`

2 The Model

We propose a "local citizen-candidate" model. The country is subdivided in a continuum of districts i. There is one national first-past-the-post election, that involves a seat attached to each district. Fundamental in our model is the fact that parties must find a local candidate to win a district.

Our candidate-selection procedure draws on two different strands of the literature. On the one hand, we follow Snyder and Ting (2002) in assuming that Nature draws candidates from some pool of candidates. Each of this potential candidate is offered to represent either party L, party R or to be an independent. Like in Snyder and Ting, we assume that benefits and costs are such that it is a dominant strategy to accept running as an independent. Conversely, only candidates that are not "too different" from the national party line will accept to run under the banner of a party P. In contrast to Snyder and Ting, however, we follow the citizen-candidate approach (Osborne and Slivinski 1996, Besley and Coate 1997) insofar as we impose that the distribution of potential candidates is the same as that of local citizens. By contrast, Snyder and Ting (2002) assume some exogenous distribution of candidates that is distinct from either national or regional citizens.

2.1 Timing

The timing is as follows: at time 0, the distribution of districts and voters in each district are determined. At time 1, observing these distributions, party L and party R select a platform. At time 2, after observing the distributions determined at time 0 and the platform of each party, each potential candidate in each district states under which banner(s) he is willing to run. Once candidates are identified, elections are held at time 3, and pay-offs are realized.

2.2 Payoffs

Voters: voters' utility is decreasing quadratically in the distance between their bliss policy, y, and the implemented policy x:

$$U(y, x) = -(y - x)^2 .$$

With probability λ, voters elect the local Condorcet winner. With probability $1 - \lambda$, they elect a candidate at random. We only consider the case $\lambda \to 1$. This assumption of $\lambda \neq 1$ allows us to discard counterintuitive equilibria in which a candidate would only run if he has a 0-probability of being elected.

Parties: Each party garners a pay-off that is proportional to the number of districts won.

Independent candidates:

$$\begin{aligned} V_I(x_j) &= w_I > 0 \text{ if elected} \\ &= 0 \text{ otherwise.} \end{aligned}$$

Party candidates: being elected under the banner of party P involves a "discipline cost", that is strictly increasing in the distance between the ideology of the candidate, x_j, and the platform of the party, x_P:

$$V_P(x_j) = w_P - \Delta(|x_P - x_j|) \text{ if elected,}$$
$$= 0 \text{ otherwise.}$$

We impose that w_P and $\Delta' > 0$, and define $\theta \equiv \Delta^{-1}(w_P)$. Party discipline, that is, the slope of the cost Δ imposed on candidates that deviate from the national party line implies that only candidates with $|x_P - x_j| \leq \theta$ accepts to run under the banner of party P, whereas potential candidates with $|x_P - x_j| > \theta$ turn down the offer. Note that the tighter is internal party discipline (i.e. the larger is Δ'), the smaller is θ.

By contrast, it is a dominant strategy to take advantage of the opportunity to run as an independent candidate, even if $w_I < w_P$.

The effect of party discipline

The result of the candidate selection process is that the political preferences of an independent candidate in a district i can be that of any local citizen. Framed differently, the policy that an independent could implement is distributed according to the same distribution as that of preferences in the district. Formally, if $f_i(y)$ is the distribution of preferences in district i, we have:

$$f_i(x_I) = f_i(y), \forall x_I \tag{1}$$

where x_I is the policy that I would implement if elected.

Conversely, a candidate j in party P is bound to have preferences within distance θ of the party's national platform:

$$f_i(x_j|x_P) = f_i(y), \forall x_P \in [x_P - \theta; x_P + \theta],$$
$$= 0 \text{ otherwise.}$$

(Ex-post) National party discipline thus helps generating more cohesive groups of candidates than without the presence of the party.

For simplicity, let us assume from now on that the distribution of citizens in a district i is uniform:

$$y \sim \mathsf{U}[y_i - 1; y_i + 1]. \tag{2}$$

From the uniform distribution, we immediately find the expected position of theindependent candidate:

$$\mathsf{E}_i(x_I) = y_i,$$

as well as that of the party candidate:

$$\mathsf{E}_i\left(x_P\right) = x_P;\ \text{if}\ \left|y_i - x_P\right| \leq 1-\theta$$

$$\mathsf{E}_i\left(x_P\right) = \frac{y_i + x_P}{2} + \frac{1-\theta}{2},\ \text{if}\ 1+\theta \geq \left|y_i - x_P\right| \in \left(1-\theta,\ 1+\theta\right].$$

This implies that:

Lemma 1 *In districts* ***close*** *to x_P, the typical party candidate has ideology x_P.*
In a district ***far left*** *from x_P, the typical party candidate has an ideology to the left of x_P but to the right of y_i.*
In a district ***far right*** *from x_P, the typical party candidate has an ideology to the right of x_P but to the left of y_i.*

`[Comment and explain]`

3 The value of a party in a district

Now, analyse whether a given district values positively or negatively the party's "branding" by checking whether the median voter prefers an independent or a candidate affiliated to a party. Going through this analysis will tell us which district seats can be won by the party and help us understanding the strategic decisions of the parties, in the next sections.

Since the voters are faced with some ex ante uncertainty regarding the policies that would be implemented by each candidate, they will base their voting decision upon the expected utility they receive from either candidate:

$$\mathsf{E}U\left(y,x\right) = -\int \left(y_i - x_j\right)^2 dF\left(x_j\right) = -\left(y_i - \mathsf{E}x_j\right)^2 - \sigma_i^2\left(x_j\right).$$

This expected utility does depend both on the expected platform $\mathsf{E}x_j$ that j would implement, and on the uncertainty surrounding its ideology. Note that, because of the selection process, this uncertainty is district-specific.

From (1) and (2), it follows that:

$$\mathsf{E}U_i\left(x_I\right) = -1/3,\ \forall i.$$

That is the expected utility that the median voter derives from electing an independent is identical in all districts.

Conversely, the expected utility derived from electing the candidate from party P depends on the distance $|y_i - x_P|$. We have two cases to consider:

- $|y_i - x_P| \leq 1 - \theta$. When distance is small, we have: $\sigma_i^2 (x_P) = \theta^2/3$, and hence:

$$\mathsf{E}U_i (x_P) = -(y_i - x_P)^2 - \theta^2/3.$$

- $|y_i - x_P| > 1 - \theta$. When distance is large, we have: $\sigma_i^2 (x_P) = \beta_i^2/3 \left(< \theta^2/3\right)$, with $\beta_i \equiv (1 + \theta - |y_i - x_P|)/2$. In this case, we have:

$$\mathsf{E}U_i (x_P) = -(\beta_i - 1)^2 - \beta_i^2/3.$$

`[Explain the effects and discuss]`

Comparing the expected utility derived from electing a party candidate or an independent tells us which districts value parties positively, depending on party discipline:

Proposition 1 *Assume 'strict' party discipline: $\theta \leq 1/2$. Then, parties are valued positively (i.e. the party candidate beats the independent) in all districts i such that: $y_i \in \left[x_P - \sqrt{\frac{1-\theta^2}{3}}; x_P + \sqrt{\frac{1-\theta^2}{3}}\right]$. Note that the size of this set is* in*creasing in party discipline: $\partial\, 2\sqrt{\frac{1-\theta^2}{3}}/\partial\theta < 0$.*
Assume 'lax' party discipline: $\theta > 1/2$. Then, parties are valued positively in all districts i such that: $y_i \in [x_P - \theta; x_P + \theta]$. The size of this set is de*creasing in party discipline: $\partial\, 2\theta/\partial\theta > 0$.*

Proof. `To be re-written`

Case 1: $y_i + 1 \geq x_L + \theta$.

$$EU_i (x_L) = -(y_i - x_L)^2 - \theta^2/3 \geq -\frac{1}{3} = EU_i (z_u).$$

Case 2: $y_i + 1 < x_L + \theta$.

$$\begin{aligned} EU_i (x_L) &= -(y_i - \mu_i (x_L))^2 - \sigma_i^2 (x_L) \\ &= -\left(y_i - \frac{1 + y_i + x_L - \theta}{2}\right)^2 - \frac{\beta_i^2}{3} \\ &= -\left(\frac{1 + y_i - x_L + \theta}{2} - 1\right)^2 - \frac{\beta_i^2}{3} \\ &= -(\beta_i - 1)^2 - \frac{\beta_i^2}{3}. \end{aligned}$$

Thus $EU_i(x_L) \geq EU_i(z_u) = -1/3$ if and only if $\beta_i \in [1,2]$ and substituting for β_i yields:

$$\begin{aligned} |x_L - y_i| &\leq \sqrt{\frac{1-\theta^2}{3}}, \ \ \forall y_i + 1 \geq x_L + \theta, \text{ or} \\ \theta - 1 &\leq |x_L - y_i| \leq \theta, \ \ \forall y_i + 1 < x_L + \theta. \end{aligned} \tag{3}$$

Thus, when $x_L - y_i > 1 - \theta$, the candidate of party L beats the unaffiliated iff $x_L - y_i$ is also smaller than θ. However, the set $[1-\theta, \theta]$ is non-empty on $\mathcal{R}^+$ iff $\theta > 1/2$.

Hence, for $\theta < 1/2$, (3) cannot be met. This implies that district i elects candidate L iff:

$$|x_L - y_i| \leq \sqrt{\frac{1-\theta^2}{3}}. \tag{4}$$

If instead $\theta \geq 1/2$, because in this case $|x_L - y_i| \leq \theta \Rightarrow |x_L - y_i| \leq \sqrt{\frac{1-\theta^2}{3}}$, L beats all the unaffiliated in all the districts such that $y_i + 1 > x_L + \theta$ ($\equiv x_L - y_i < 1 - \theta$) and beats the unaffiliated in the districts where $y_i + 1 < x_L + \theta$ if and only if $|x_L - y_i| \leq \theta$. In other words, for $\theta \geq 1/2$, the relevant condition is:

$$|x_L - y_i| \leq \theta. \tag{5}$$

■

`[Explain trade-off between discipline, capacity to adapt to local preferences, and signal quality]`

4 The parties' locational strategy

Now that we have identified the link between the platform of a party and the voters' support for the party in each district, we are in a position to analyze the parties' optimal strategy.

Clearly, the optimal strategy of the party ought to depend on the distribution of districts in the country. For simplicity, we assume that districts are uniformly distributed: $y_i \sim \mathcal{U}[-a, a]$, with $a > 0$.

`[Write down extended results]`

First, we analyze how a single national party should behave when faced with a myriad of local parties (independents). The objective of the party is to maximize its vote share (or the number of seats won), S_P. To ensure that we always have an internal solution, we set $a > 1$:

Proposition 2 *Assume districts are uniformly distributed:* $y_i \sim \mathcal{U}[-a,a]$, *with* $a \geq 1$, *and that there is only one national party. Then,* $0 \in \arg\max_{x_P}(S_P)$. *That is* $x_P^* = 0$ *weakly dominates all other platform positions. The vote share of the national party is then equal to:*

$$\begin{aligned} S_P &= \sqrt{\frac{1-\theta^2}{3a^2}}, \text{ if } \theta < 1/2; \\ &= \frac{\theta}{a}, \text{ if } \theta \in (1/2,\ 1). \end{aligned}$$

Proof. For any θ and any platform x_L, the party's vote share is a function on the position of x_L in $[-a,a]$. Given that district medians are uniformly distributed, this vote share is maximized for any x_L^* such that $[x_L^* - \theta; x_L^* + \theta] \in [-a,a]$. One such platform is $x_L^* = 0$. In this case, the vote share is given by $\Pr\left[|0 - y_i| \leq \max\left[\sqrt{\frac{1-\theta^2}{3}}, \theta\right]\right]$, which immediately yields Proposition 2. ■

Note also that S_L is strictly increasing in θ for $\theta > 1/2$. Therefore, the closer θ is to 1, the larger is the number of districts won by the party.

Comparison with Snyder and Ting (2002)

This proposition is worth comparing to the main result in Snyder and Ting (2002). In their model, $x_L = 0$ strictly dominates all the other positions because the national party then offers the same platform as any independent, though with a smaller variance. This position ensures that L beats unaffiliated candidates in *all districts*, as long as the party does perform some selection ($\theta < 1$).

In our model, and in contrast to Snyder and Ting's, a lower θ also implies that the party has a reduced ability to respond to local preferences, which reduces the "marketability" of the party brand name. This is why a lower θ may either decrease or increase the vote share of the national party. As will be seen below, it is also worth stressing that if $x_P^* = 0$ is an optimal platform, this is only because the party is alone. In Snyder and Ting, two parties that compete with one another generally find it optimal to both locate in 0 (unless discipline is extremely lax).

As shown below, this result is not robust to the introduction of local candidates that can be district-specific. Platform polarization is a natural outcome whenever there is a conflict between strictly imposing the national party line onto local candidates, and their ability to adapt to local preferences to compete with the independent candidates.

4.1 Optimal polarization

If there is a second party, there can be districts in which either party candidate beats an unaffiliated candidate. Potentially, both parties could win in such districts. We thus have to check which of the candidates obtains the support of the median voter in the district.

Let parties L and R be defined by their ideological positions: $x_L \leq x_R$. The set of districts in which both parties could win is defined by Proposition 1. Again, we have to distinguish two cases.

4.2 $\theta \leq 1/2$

If $\theta \leq 1/2$, a party may win against an independent if and only if $y_i + 1 > x_P + \theta$, which implies in turn that $\sigma_i^2 (x_P) = \frac{\theta^2}{3}$.

Let: $\kappa(\theta) \equiv \sqrt{\frac{1-\theta^2}{3}}$. Proposition 1, we know that any party P beats unaffiliated candidates in all districts i such that: $y_i \in [x_P - \kappa(\theta), x_P + \kappa(\theta)]$.

Hence, the set of districts in which both parties could run is:

$$y_i \in [x_R - \kappa(\theta), x_L + \kappa(\theta)] \text{ if this set is non-empty.}$$

In the present case ($\theta \leq 1/2$), the expected utility of the median voter is:

$$\begin{aligned} EU_i(x_L) &= -(y_i - x_L)^2 - \frac{\theta^2}{3} \text{ if candidate } L \text{ is elected} \\ EU_i(x_R) &= -(y_i - x_R)^2 - \frac{\theta^2}{3} \text{ if candidate } R \text{ is elected.} \end{aligned}$$

Hence, L beats R iff:

$$|y_i - x_L| < |y_i - x_R|.$$

Lemma 2 *If districts are uniformly distributed: $y_i \sim \mathcal{U}[-a, a]$, with $a \geq 1$, then the vote share of party L is:*

$$\begin{aligned} S_L &= \frac{x_L + \kappa(\theta)}{2a} + \frac{1}{2}, \text{ if } x_L < x_R - 2\kappa(\theta) \text{ and } x_L < \kappa(\theta) - a \\ &= \frac{x_R + x_L}{4a} + \frac{1}{2}, \text{ if } x_L > x_R - 2\kappa(\theta) \text{ and } x_L < \kappa(\theta) - a \\ &= \frac{x_R - x_L + 2\kappa(\theta)}{4a}, \text{ if } x_L > x_R - 2\kappa(\theta) \text{ and } x_L \geq \kappa(\theta) - a \\ &= \frac{\kappa(\theta)}{a}, \text{ if } x_L < x_R - 2\kappa(\theta) \text{ and } x_L \geq \kappa(\theta) - a \end{aligned}$$

The vote share of party R is:

$$\begin{aligned} S_R &= \frac{1}{2} - \frac{x_R - \kappa(\theta)}{2a}, \text{ if } x_L < x_R - 2\kappa(\theta) \text{ and } x_R > a - \kappa(\theta) \\ &= \frac{1}{2} - \frac{x_L + x_R}{4a}, \text{ if } x_L > x_R - 2\kappa(\theta) \text{ and } x_R > a - \kappa(\theta) \\ &= \frac{x_R - x_L + 2\kappa(\theta)}{4a}, \text{ if } x_L > x_R - 2\kappa(\theta) \text{ and } x_R \leq a - \kappa(\theta) \\ &= \frac{\kappa(\theta)}{a}, \text{ if } x_L < x_R - 2\kappa(\theta) \text{ and } x_R \leq a - \kappa(\theta) \end{aligned}$$

Thus each party faces two incentives, which can be ranked in terms of return to districts won:

1) each party has an incentive to move away from the bounds of the distribution of district medians (a one unit move away from those bounds entails an equal size increase in districts won);

2) given that 1) is fulfilled, each party has also an incentive to remain polarized enough to ensure that the range of politician types willing to join the two parties do not overlap (a one unit increase in the degree of polarization increases the number of districts won by 1/2).

As a consequence, we have:

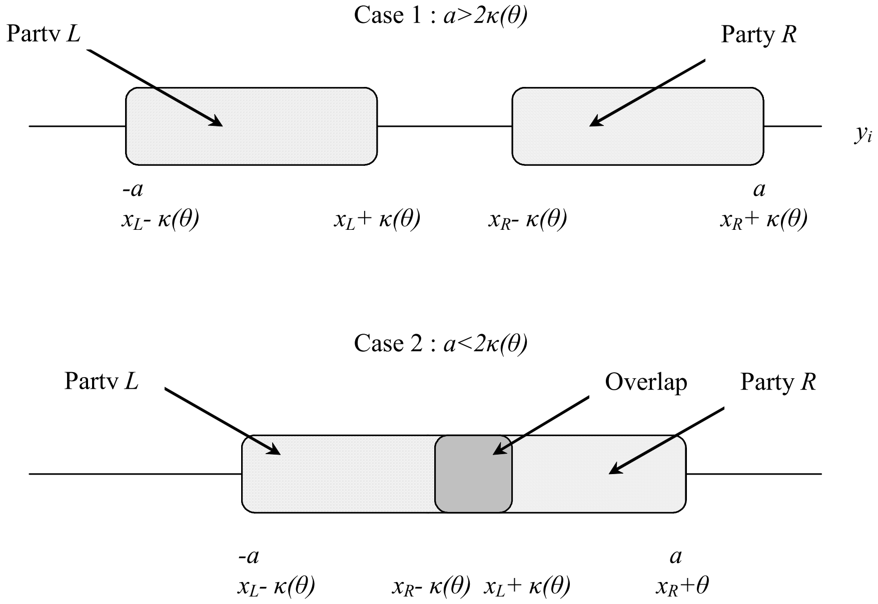
Proposition 3 *Assume $\theta \leq 1/2$. Then, equilibrium polarization $(x_R^* - x_L^*) \in [2\kappa(\theta^*), 2[a - \kappa(\theta^*)]]$ if $a \geq 2\kappa(\theta)$, and is equal to $2[a - \kappa(\theta^*)]$ if $a < 2\kappa(\theta)$.*

Proof. By Lemma 2, S_L is strictly increasing in x_L, $\forall x_L < \kappa(\theta) - a$. Therefore, $x_L < \kappa(\theta) - a$ cannot be an equilibrium. By the same token, $x_R > a - \kappa(\theta)$ cannot be an equilibrium either.

Assume $a < 2\kappa(\theta)$. In that case, we have: $\{x_L, x_R\} = \{\kappa(\theta) - a, a - \kappa(\theta)\} \Rightarrow x_L > x_R - 2\kappa(\theta)$. Hence, S_L is strictly decreasing in x_L, $\forall x_L \geq \kappa(\theta) - a$, and S_R is strictly increasing in x_R, $\forall x_R \leq a - \kappa(\theta)$. Hence, the unique equilibrium is $\{x_L^*, x_R^*\} = \{\kappa(\theta) - a, a - \kappa(\theta)\}$.

If instead $a \geq 2\kappa(\theta)$, then $\partial S_L / \partial x_L = \partial S_R / \partial x_R = 0$, $\forall\ (x_R^* - x_L^*) > 2\kappa(\theta)$. Hence, parties are indifferent between all platform positions $\{x_L, x_R\} \in [\kappa(\theta) - a, a - \kappa(\theta)]^2$ provided that $|x_R^* - x_L^*| > 2\kappa(\theta)$. ■

Graphically, the equilibrium can be represented as follows:



4.3 $\theta > 1/2$

If $\theta > 1/2$, lemma 2 tells us that party $P \in \{L, R\}$ beats unaffiliated candidates in all districts i such that $y_i \in [x_P - \theta, x_P + \theta]$.

The set of "close districts", in which both parties could run is:

$$y_i \in \mathcal{C} \equiv [x_R - \theta, x_L + \theta] \text{ if this set is non-empty.}$$

For $\mathcal{C} \neq \varnothing$, we have to compute the share of districts won by L and by R for any possible pair of platform positions.

In the previous case, i.e. for $\theta \leq 1/2$, the median voter in a given district i preferred a given party, say L, only if x_L was closer to his bliss point than x_R. In the present case, i.e. for $\theta > 1/2$, the median voter may prefer party L in 3 distinct cases:

1. x_L is closer to his bliss point than x_R and $\sigma_i^2(x_L) \leq \sigma_i^2(x_R)$. That is, the median voter prefers both the (average) ideology and the signalling content offered by party L.
2. x_L is closer to his bliss point than x_R but $\sigma_i^2(x_L) > \sigma_i^2(x_R)$. If the median voter prefers sufficiently strongly the (average) ideology offered by party L, he will not be seduced by the higher signalling content of party R.

3. x_L is farther from his bliss point than x_R but $\sigma_i^2(x_L) < \sigma_i^2(x_R)$. This case is complementary to case 2: the signalling content offered by party L is sufficiently strong to dominate its worse average ideology.

Identifying the subset of districts in $\mathcal{C}$ that prefer L to R or otherwise is therefore complex. Which districts belong in this subset depends on the exact shape of the voters' utility function and may, or may not, be a compact. Still, we can compute the vote share of each party without having to explicitate $EU_i(x_L)$ and $EU_i(x_R)$:

Lemma 3 *Assume $\theta > 1/2$ and $\mathcal{C} \equiv [x_R - \theta, x_L + \theta]$ is non-empty. Then, the vote share of party L is:*

$$\begin{aligned} S_L &= \int_{x_L-\theta}^{\frac{x_L+x_R}{2}} f(y_i)\,dy_i = \frac{1}{2a}\left(\frac{x_L+x_R}{2} - \max[-a, x_L-\theta]\right) \\ &= \frac{2\theta + x_R - x_L}{4a} \text{ if } x_L > \theta - a \\ &= \frac{1}{2} + \frac{x_L+x_R}{4a} \text{ if } x_L \leq \theta - a \end{aligned}$$

and the vote share of party R is:

$$\begin{aligned} S_R &= \int_{\frac{x_L+x_R}{2}}^{x_R+\theta} f(y_i)\,dy_i = \frac{1}{2a}\left(\min[a, x_R+\theta] - \frac{x_L+x_R}{2}\right) \\ &= \frac{2\theta + x_R - x_L}{4a} \text{ if } x_R + \theta < a \\ &= \frac{1}{2} - \frac{x_L+x_R}{4a} \text{ if } x_R + \theta \geq a \end{aligned}$$

Proof. We know that $EU_i(x_L) = f(|y_i - x_L|)$ and $EU_i(x_R) = f(|y_i - x_R|)$. These two functions are identical and symmetric around y_i, since only the distance between y_i can x_P matters.

Remember that $\theta < 1 \leq a$. Assume $x_L < 0 < x_R$. Then, if we consider the set of y_i's: $[x_R - \theta, x_L + \theta]$, symmetry tells us that half of them support party L and the other half supports party R. Outside the set $\mathcal{C}$, all districts in $[x_L - \theta, x_R - \theta)$ prefer L and the ones in $(x_L + \theta, x_R + \theta]$ prefer R. Thus, the set of districts supporting L has the same measure as the compact set:[1]

$$\bar{y}_i \in \left[x_L - \theta, \frac{x_L + x_R}{2}\right].$$

Similarly, the set of districts supporting R has the same measure as:

$$\bar{y}_i \in \left[\frac{x_L + x_R}{2}, x_R + \theta\right].$$

■

[1] We thank Catherine Dehon for suggesting this reordering of districts.

Proposition 4 *Assume that* $\theta > 1/2$*. Then, equilibrium polarization* $x_R^* - x_L^*$ *is at least as large as* 2θ *if* $a \geq 2\theta$ *and is equal to* $2[a-\theta]$*, if* $a < 2\theta$*.*

Proof. By Lemma 3, for any x_L, x_R such that $x_R - \theta < x_L + \theta$, we have:

$$\begin{aligned}\frac{\partial S_L}{\partial x_L} &> 0, \text{ if } x_L < \theta - a \\ &< 0, \text{ if } x_L > \theta - a\end{aligned}$$

$$\begin{aligned}\frac{\partial S_R}{\partial x_R} &< 0, \text{ if } x_R > a - \theta \\ &> 0, \text{ if } x_R < a - \theta\end{aligned}$$

Thus, party L strictly prefers locating in $x_L^* = \theta - a$ and party R in $x_R^* = a - \theta$ if, for that pair of locations, we have: $x_R - \theta < x_L + \theta$. That is if:

$$\begin{aligned}a - 2\theta &< 2\theta - a, \text{ or} \\ a &< 2\theta.\end{aligned}$$

If instead $x_R - \theta > x_L + \theta$ for $\{x_L, x_R\} = \{\theta - a, a - \theta\}$, we have:

$$\begin{aligned}S_L &= \int_{x_L-\theta}^{x_L+\theta} f(y_i)\, dy_i = \frac{1}{2a}\left(x_L + \theta - \max\left[-a, x_L - \theta\right]\right) \\ &= \frac{\theta}{a} \text{ if } x_L > \theta - a \\ &= \frac{1}{2} + \frac{x_L + \theta}{2a} \text{ if } x_L \leq \theta - a\end{aligned}$$

and similarly:

$$\begin{aligned}S_R &= \frac{\theta}{a} \text{ if } x_R < a - \theta \\ &= \frac{1}{2} + \frac{\theta - x_R}{2a} \text{ if } x_R \geq a - \theta\end{aligned}$$

which implies that: $\partial S_L / \partial x_L = 0 = \partial S_R / \partial x_R$ for any pair of positions $\{x_L, x_R\}$ such that:

$$x_L \geq \theta - a, \; x_R \leq a - \theta \text{ and } x_R - x_L \geq 2\theta. \tag{6}$$

Hence, if the dispersion of ideologies across districts is sufficiently large compared to party discipline ($a \geq 2\theta$), then (6) is a necessary and sufficient equilibrium condition. ■

5 Endogenous choice of discipline

TBW

6 Conclusion

TBW

7 References [incomplete]

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