Notes for a revision of the a priori voting power paradigm*
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Abstract

In this paper we make some proposals for a more clear organization of knowledge in voting power theory. We propose a basic distinction between two scenarios in which a committee can make decisions under a voting rule. In a 'take-or-leave-it' scenario the committee has only the capacity of accepting or rejecting by vote proposals submitted externally to it, while in 'bargaining' committee negotiation is feasible and the voting rule conditions it by imposing which groups of voters can enforce any agreement. From this double point of view we reinterpret and revise the foundations and normative recommendations of voting power theory.

Key words: Collective decision-making, voting power theory, power indices

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1 Introduction

In the last years, a renewal of interest in voting power issues has given rise to a conspicuous increase in the number of academic papers, both theoretical and applied, related by one way or other to what can be tentatively called voting power theory[1]. No doubt the important changes that have taken place in the EU as a result of the last enlargements, and the necessity of redesigning the decision making procedures have contributed to it. This has also given rise to heated debates within the scientific community, and movements within it to go beyond the academic realm. As a result there have been different attempts to influence politicians or their advisers, claiming as legitimacy for it a supposedly well established body of knowledge. A rather active group has been the one adhering to what we call here, borrowing Kuhn's (1962) term, the dominant a priori voting power "paradigm", based on the power indices' tradition. The lack of consensus among scholars even about the basic models and their relevance may have been one of the reasons of the relative failure of this endeavour. But our purpose here is not to survey the discrepancies of the competing approaches. The purpose of this paper is to provide some explicit suggestions for a "shift" in search of a richer and more convincing paradigm still within the a priori point of view but widening the conceptual framework.

What we call here the current a priori voting power "paradigm"[2] is a terminology and a way of articulating knowledge, shared by a certain number of researchers within the academic world, related to the power indices literature, which is basically surveyed and systematized in Felsenthal and Machover's (1998) book. In fact, this book can be seen as the closest to an embodiment of the paradigm. But our purpose in not criticize it but to provide a clear and wider conceptual framework.

The rest of the paper is organized as follows. In section 2, we give summarily our way of approaching the question and convey the main ideas. In the next two sections we concentrate on each of the two basic scenarios in which a committee may have to make decisions. In section 3 we consider *'take-it-or-leave-it'* situations, and in section 4 we consider *'bargaining committees'* situations. In section 5 we summarize the main conclusions.

[1] We give up a cumbersome or even a summary list of all or main contributions in the field as unnecessary for the purpose of this paper.

[2] A bit exaggeratedly in fact. In Khun's terminology it would be possibly more adequate to refer to it as a "preparadigm", given the limited consensus about it. Nevertheless, risking the philosophers of science's criticism, we find the term suggestive and aproximately adequate.

2 Some suggestions for a 'shift'

The specification of a voting rule to make dichotomous choices ('yes'/'no', acceptance/rejection) does not involve nor requires the description of its users or 'players'. It is enough to specify the vote configurations that would mean acceptance and those that would mean rejection. A same voting rule can be used by different sets of users to make decisions about different types of issues. But describing a voting rule as a simple TU game makes the trick of producing a game where there were no players. Then applying to it the recently introduced Shapley (1953) value the Shapley-Shubik (1954) index was born. Shapley and Shubik interpret it as an evaluation of a priori 'voting power' in the committee. As the marginal contribution of a player to a coalition in this game can only be 0 or 1, being 1 only when the presence/absence of a player in a coalition makes it winning/losing, they also propose an interpretation in terms of likelihood of being pivotal or decisive. Hence the seminal duality or ambiguity:

Q.1: *The Shapley-Shubik index is a 'value', that is, an expected payoff of a sort of bargaining situation, or an assessment of the likelihood of being decisive?*

Later Banzhaf (1965) adheres consistently the point of view of power as decisiveness, and criticizes the Shapley-Shubik index in view of the unnatural probability model underlying its probabilistic interpretation in the context of voting. If power meant being decisive, then a measure of power is given by the probability of being it. Thus an a priori evaluation of power, if a priori all vote configurations are equally probable, is the probability of being decisive under this assumption. (Independently, Penrose in 1946 reached basically the same conclusion in a narrower formal setting.) In fact Banzhaf only 'almost' says so for he destroys this clean probabilistic interpretation by 'normalizing' this vector. So the old dispute is served:

Q.2: *Which is better as a measure of voting power: the Shapley-Shubik index or the Banzhaf index? What are more relevant axioms or probabilities?*

It is our view that in order to solve these dilemmas and dissipate ambiguities a more basic issue is to be addressed: *What are we talking about?* In lieu of proceeding from abstract terms as 'power' related to an exceedingly broad class of situations (any collective body that makes decisions by vote) and getting entangled prematurely with big words, we think it wiser to

(i) *start by setting the analysis of voting situations as the central goal.*

Collective decision-making by vote may include an extremely wide and heterogeneous constellation of voting situations: a parliament law-making, a parliament vote for the endorsement of a government, a referendum, a presidential election, a governmental cabinet decision-making, a shareholders' meeting, an international or intergovernmental council, and a long etcetera. By setting the analysis of voting situations as the central goal, instead of the abstraction 'voting power', we mean

(ii) *starting from clear-cut models of well specified clear-cut voting situations,*

instead of starting from words denoting hardly specified abstractions in vaguely specified situations. For instance, it is not the same a committee with capacity to bargain the proposal before voting, that one only allowed to accept or reject proposals by vote. It is not the same millions of voters than a few, etc.

A dichotomy (open to become part of a wider 'multichotomy'), consistent with the above principles should distinguish between two types of voting situations or committees which make decisions under a voting rule: 'take-it-or-leave-it' committees vs. 'bargaining' committees.

(ii.1) A *'take-it-or-leave-it' committee* votes upon different independent proposals over time, submitted to the committee by some external agency, so that the committee is only entitled to accept or reject proposals, but cannot modify them.

(ii.2) A *'bargaining' committee* deals with different issues over time, so that for each issue a different configuration of preferences emerges among its members over the set of feasible agreements, and the committee bargains about every issue in search of a consensus, in search of which is entitled to adjust the proposal.

Though in reality it is often the case that a same committee acts sometimes like a 'take-it-or-leave-it' committee, and others like a 'bargaining' committee, or even something between, this crisp differentiation of two clear-cut types of situation provides benchmarks for a better understanding of many mixed real world situations. As we will see in the next two sections, they require different models and raise different questions with different answers which give rise to different recommendations. This neat differentiation and the analysis therein will also shed some light in the old ambiguities.

A common ingredient, though, in both types of committee is the voting rule that governs decisions. In order to proceed a minimum of notation is needed. If n voters, labelled by $N = \{1, 2, .., n\}$ are asked to vote 'yes' or 'no' on an issue, any result of a vote, or *vote configuration,* can be summarized by the subset of voters who vote 'yes': $S \subseteq N$. An *N-voting rule* is then specified by a set $W \subseteq 2^N$ of *winning* vote configurations such that (i) $N \in W$; (ii) $\emptyset \notin W$; (iii) If $S \in W$, then $T \in W$ for any T containing S; and (iv) If $S \in W$ then $N \backslash S \notin W$.

3 'Take-or-leave-it' committees

3.1 The 'take-or-leave-it' environment

A pure 'take-it-or-leave-it' environment is that of a committee that makes dichotomous decisions (acceptance/rejection) by vote under the following conditions: (i) the committee votes upon different independent proposals over time; (ii) proposals are submitted to the committee by some external agency; (iii) the committee is only entitled to accept or reject proposals, but cannot modify them; and (iv) a proposal is accepted if a winning vote configuration according to the specifications of the voting rule emerges.

In real world committees rarely all these conditions are strictly satisfied. Nevertheless, for a sound analysis it is necessary to make explicit and precise assumptions about the environment, and this is the only way to have clear conclusions. Indeed, traditional power indices and voting power theory credibility is mined by the lack of clarity about the precise specification of the underlying collective decision-making situation.

Under the above conditions, it seems clear that unless the case of indifference about the outcome each voter's vote is determined by his/her preferences. In particular these conditions leave no margin for bargaining. The impossibility of modifying proposals, their independence across time, etc., rules our the possibility of bargaining and consequently of strategic behavior. In other words, *in a pure 'take-it-or-leave-it' environment game-theoretic considerations are out of place.*

3.2 The basic issues in 'take-or-leave-it' environment

As briefly commented in the introduction, at the foundations of traditional voting power theory there are some ambiguities concerning the precise conditions under which the collective decision-making process takes place, as well as about the interpretation of some power indices. Either explicitly or implicitly the notion of power as decisiveness, i.e. the likelihood of one's vote being in a position to decide the outcome, underlies most of traditional voting power literature.

Were it not for the weight of theoretical inertia it would hardly be necessary to argue about the irrelevance of this notion in a pure 'take-it-or-leave-it' environment, where voting behavior follows trivially preferences[3]. *Decisiveness may be a form or, more precisely, a source of power only in a situation in which there is room for negotiation and the*

[3]Rae (1969), Brams and Lake (1978), Barry (1980) (from whom we take the term of "success"), Straffin, Davis, and Brams (1981), and more recently König and Bräuninger (1998) have paid attention to the notion of success or satisfaction, but the ambiguity has kept unsolved by the lack of definite clarification about the underlying voting situation. In Laruelle, Martínez and Valenciano (2004) we argue in support of success as the relevant notion in a 'take-it-or-leave-it' committee.

possibility of using it with this purpose. But the conditions that specify a 'take-it-or-leave-it' environment preclude such possibility. For instance, voting on an issue against one's preferences in exchange of someone else doing the same in one's favor on a different issue is not possible in case of strict independence between proposals, as assumed.

The interest of any voter is having the desired outcome, and in a 'take-it-or-leave-it' situation nothing better can be done to that end than just voting accordingly. Thus, *having success or satisfaction is the central issue in this kind of voting situation.* If so the assessment of a voting situation of this type with normative purposes requires assessing the likelihood of each voter having his/her way. *For an a priori assessment it seems natural to assume all preference configurations or, equivalently (at least if no indifference occurs), all vote configurations equally probable.* This is the assumption that underlies a variety of power indices in the literature: the probability of every vote configuration S is $\frac{1}{2^n}$.

Assuming this probabilistic model, the probability of a voter i being successful in a vote under a voting rule W is given by

$$Succ_i(W) := Prob\ \ (i \text{ is successful}) = \sum_{S:i\in S\in W} \frac{1}{2^n} + \sum_{S:i\notin S\notin W} \frac{1}{2^n}. \tag{1}$$

As commented above, more attention as been paid to the probability of being decisive under the same probabilistic model, given by

$$Dec_i(W) := Prob\ \ (i \text{ is decisive}) = \sum_{\substack{S:i\in S\in W \\ S\backslash i\notin W}} \frac{1}{2^n} + \sum_{\substack{S:i\notin S\notin W \\ S\cup i\in W}} \frac{1}{2^n}. \tag{2}$$

Other conditional variants can be considered. The following table (from Laruelle and Valenciano, 2005a) presents the probabilities of being either successful or decisive unconditioned and conditioned for different conditions, and the index that gives it: Rae (1969), Banzhaf (1965), Coleman's (1971, 1986) indices to prevent ($Col(P)$) and to initiate ($Col(I)$) action, König and Bräuninger's (1998) inclusiveness index (K-B).

Condition:	*none*	*i votes 'yes'*	*i votes 'no'*	*acceptance*	*rejection*
Success	*Rae*	?	?	*K-B*	?
Decisiveness	*Banzhaf*	*Banzhaf*	*Banzhaf*	$Col(P)$	$Col(I)$

Table 1

The question marks "?" mean that no index in the literature has so far paid attention to these probabilities. In the case of the probability of success conditional to the proposal being rejected, this is the natural pair of König and Bräuninger's (1998) inclusiveness index

(K-B). As to the probabilities of a voter being successful *conditional* either to his/her voting 'yes' or to his/her voting 'not', in Laruelle, Martínez, and Valenciano (2004) where we argue in favor of the success as the relevant notion in a 'take-it-or-leave-it' environment, these probabilities are calculated for the voting rules in the Council of Ministers of the EU with interesting results[4].

3.3 Normative recommendations

If the relevant issue in a 'take-it-or-leave-it' environment is that of the likelihood of success, then this should be the base for normative recommendations. The question arises of what recommendations can be made for the choice of the voting rule in a committee of this type in which every member acts on behalf of a group of different size. There are two basic points of view to base such recommendations: egalitarian and utilitarian. The implementation of either of the two principles with respect to the represented people requires some idealization of the influence of these individuals on the decisions of the committee. The well known idealized two-stage decision process assuming that each representative follows the majoritarian opinion of his/her group on every issue can be neatly modeled by a composite rule in which decisions are made directly by the represented people. The recommendations that result from either point of view in the literature based on the Banzhaf index (therefore in the notion of power as decisiveness) are the 'square root rule' (Penrose (1946)) and the 'second square root rule' (conjectured by Morriss (1987) and rigorously proved by Felsenthal and Machover (1999)).

In view of the linear relation that exists between the probability of being decisive and that of being successful when all vote configurations are equally probable[5], it can be seen that founding the choice on either principle but basing it on the notion of success yields the same recommendations.

Square root rule for a take-it-or-leave-it committee: The egalitarian principle, according to which all individuals should have an equal probability of success, would be (approximately and under certain conditions) implemented by a voting rule in the committee for which the Banzhaf index of each representative were proportional to the

[4] A natural extension of (1) and (2), as well as of any entry on Table 1 is possible replacing this particular probability distribution over voting configuration by an arbitraty one (see Laruelle and Valenciano (2005a)). This perspective is also adopted in Laruelle and Valenciano (2005b) to give a clear account of some 'voting power paradoxes' and showing the lack of discriminating power of the postulates behind some of them, even to distinguish success and decisiveness, and in Laruelle, Martínez and Valenciano (2004) to show the analytical differences between success and decisiveness

[5] In Laruelle, Martínez, and Valenciano (2004) it is shown how this relation holds only for this particular probability distribution.

square root of the size of the group s/he represents.

Similarly the utilitarian principle yields the

Second square root rule for a take-it-or-leave-it committee: The utilitarian principle, according to which the sum of all individuals probability of success should be maximized, would be (approximately and under suitable conditions) implemented by a weighted voting rule that assigns to each representative a weight proportional to the square root of the group sizes, and the quota equals half the sum of the weights.

Nevertheless it is worth remarking that the underlying justifications are different and these differences are important. *Indeed, such recommendations have a clear justification based on either principle only for this special type of committee, while for the more complex case of bargaining committees such recommendations lack a clear basis.* The lack of a precise specification of the situation in the traditional analysis has kept hidden this important point so far. As we will see in bargaining committee these recommendations lack foundations and this has important consequences for the applications.

4 Bargaining committees

4.1 The bargaining environment

A bargaining committee also makes decisions under a voting rule, but under completely different conditions. In a bargaining committee: (i) The committee deals with different issues over time; (ii) for every issue a different configuration of preferences emerges among the members of the committee over the set of feasible agreements concerning the issue at stake; (iii) the committee bargains about every issue in search of a consensus, in search of which is entitled to adjust the proposal; and (iv) any winning coalition can enforce any agreement.

Now the situation is much more complicated. The environment permits bargaining among the members of the committee, and consequently behavior does not follow trivially preferences any more. *Now game-theoretic considerations are in order because the situation is inherently game-theoretic.*

4.2 The basic issues in bargaining environment

First note that *in a bargaining situation the basic issue is that of the outcome* of negotiations. That is, given a preference profile of the members of the committee and a bargaining environment, what will the outcome be? Or at least, what outcome is reasonable to expect? It should be remarked that only based on an answer to this basic question, others issues also relevant may be addressed. For instance, the question of the influence of the voting rule on the outcome of negotiations, or the question of the 'power' that the voting rule gives to each member. In particular, the meaning of the term 'power' in this context can only be clear when one has an answer for the first basic question.

In order to provide an answer to the central question a formal model of a bargaining committee is needed. The minimal information that a model of bargaining committee should incorporate is: the voting rule under which negotiation takes place, and the players's (now the usual game-theoretic term is adequate) preferences. As will be commented later other elements should be included for a more realistic model, but it is convenient to start with as simple as possible a model to see what conclusions can be drawn from it. In Laruelle and Valenciano (2004a) we introduce a model of a n-person bargaining committee incorporating these two ingredients. The first element is just the n-person voting rule W. As to the preferences, under the same assumptions as in Nash (1950), i.e. assuming the the players' preferences are expected utility or von Neumann-Morgenstern (1944) preferences, can be represented à la Nash by a pair $B = (D, d)$, where D is the set[6] of feasible utility vectors or 'payoffs', and d is the vector of utilities in case of disagreement.

In this two-ingredient setting the question of the rational expectations about the outcome of negotiations can be addressed from two different game-theoretic points of view: the cooperative and the non cooperative approach. The cooperative method consists of ignoring details concerning the way in which negotiations take place, and 'guessing' the outcome of negotiations between ideally rational players by means of assuming reasonable properties of the map that maps 'problems' (B, W) into payoffs $\Phi(B, W)$. The best and perhaps most influential paradigm of the cooperative approach is Nash's (1950) bargaining solution. In Laruelle and Valenciano (2004a) it is Nash's classical approach that we have imitated in our two ingredient setting. *We assume that players in a bargaining committee bargain in search of unanimous agreement.* In this way, by assuming adequate adaptation to our setting of some reasonable conditions to expect for a bargaining outcome (efficiency, anonymity, independence of irrelevant alternatives, invariance w.r.t. affine transformations, and null player), it is proved that a general 'solution' (i.e. a map $(B, W) \longmapsto \Phi(B, W)$) should be of the form

$$\Phi(B, W) = Nash^{\varphi(W)}(B) = \arg\max_{x \in D_d} \prod_{i \in N} (x_i - d_i)^{\varphi_i(W)}. \tag{3}$$

That is to say: a reasonable outcome of negotiations is given by the weighted Nash bargaining solution (Kalai, 1977) where the weights are a function $\varphi(W)$ of the voting rule,

[6] D is a *closed, convex* and *comprehensive* (i.e., $x \leq y \in D \Rightarrow x \in D$) set containing d, such that there exists some $x \in D$ s.t. $x > d$, and such that the set $D_d := \{x \in D : x \geq d\}$ is *bounded*.

moreover this function must satisfy anonymity and null-player. And note that these two properties are the most compelling ones concerning power indices[7]. *Thus formula* (3) *sets the 'contest' between power indices candidates to replace* $\varphi(W)$ *in* (3) *in a new setting and provides a new interpretation of them in terms of 'bargaining power' in the precise game theoretic sense.* In the same paper we show how adding an adaptation of Dubey's (1975) lattice property to the other conditions the Shapley-Shubik index is singled out in (3), that is, in this case the solution is

$$\Phi(B,W) = Nash^{Sh(W)}(B) = \arg\max_{x \in D_d} \prod_{i \in N} (x_i - d_i)^{Sh_i(W)}. \tag{4}$$

It is also interesting to remark that, as shown in Laruelle and Valenciano (2004a), when the bargaining element, that is to say, the preference profile in the committee summarized by $B = (D, d)$ is *transferable utility* like, that is, $B = \Lambda := (\Delta, 0)$, where $\Delta := \left\{ x \in R^N : \sum_{i \in N} x_i \leq 1 \right\}$, then we have that (3) and (4) become respectively:

$$\Phi(\Lambda, W) = Nash^{\varphi(W)}(\Lambda) = \bar{\varphi}(W) \tag{5}$$

$$\Phi(\Lambda, W) = Nash^{Sh(W)}(\Lambda) = Sh(W), \tag{6}$$

where $\bar{\varphi}(W)$ denotes the normalization of vector $\varphi(W)$.

As we said above there is also the noncooperative point of view. In the noncooperative approach the model should specify with some detail the way in which negotiations take place. This is not simple nor obvious in a situation of which the only ingredients so far are the voting rule that specifies what sets of members of the committee have the capacity to enforce any agreement, and the voters' preferences. A noncooperative modeling has necessarily to choose a 'protocol' to reach any conclusion. The question arises whether (3) or (4) have a noncooperative foundation. In Laruelle and Valenciano (2005c) we address this problem and provide noncooperative foundations to (3) and (4). Assuming complete information, we model a family of non cooperative bargaining protocols based on the voting rule that provides non cooperative foundations for (3), which appear in this light as limit cases. The results based on the noncooperative model evidence the impact of the details of protocol on the outcome, and explains the lack of definite arguments (i.e. of compelling axioms beyond argument) to go further than (3). Nevertheless (4) emerges associated with a very simple protocol also based on the voting rule under which negotiations take place, thus providing some sort of focal appeal as reference term to the Shapley-Shubik index as an a priori measure of bargaining power.

[7]These properties are satisfied by the two most popular power indices, but also by any semivalue (Dubey, Neyman and Weber (1981), see also Laruelle and Valenciano (2001, 2002, 2003)).

4.3 Normative recommendations

The model summarized in the previous section can be taken as the base for addressing the normative question of the most adequate or fair voting rule in a committee of representatives. Namely, if a voting rule is to be chosen for a committee that is going to make decisions in a bargaining environment: what is the adequate voting rule if each member will act on behalf of a group of different size?

In Laruelle and Valenciano (2004b) we address this problem. The question is tricky because for each issue a different configuration of preferences will emerge in the population represented by the members of the committee. Thus if by 'adequate' we mean fair in some sense, nothing can be said unless some form or other of relation about the preferences within each group is assumed. By 'fair' we understand *neutral* in the following sense. A neutral voting rule for the committee is such that all people represented see as indifferent direct bargaining (ideal and unfeasible, but theoretically tractable according to Nash classical bargaining solution) and leaving it in the hands of a committee of representatives. This is obviously utopian, but it can be proved to be implementable under some ideal symmetry conditions about the preferences within each group. In real world situations this condition may well fail to occur in most cases, but this idealization seems a reasonable term of reference if a voting rule is to be chosen. In fact assuming certain conditions of symmetry within each group the following recommendation arises:

A neutral voting rule (Laruelle and Valenciano, 2004b): *A neutral voting rule in a bargaining committee of representatives is one that gives each member a bargaining power proportional to the size of the group he/she represents.*

Note that this recommendation is based on (3), that is to say, does not presupposes (nor there is need of it) which is the right φ in formula (3), but mark all the same the difference with the square root rule recommendation. The neutral voting rule would be one for which:

$$\frac{\varphi_i(W)}{m_i} = \frac{\varphi_j(W)}{m_j} \qquad (\forall i, j \in N),$$

while according the square root rule the fair voting rule is one for which

$$\frac{Bz_i(W)}{\sqrt{m_i}} = \frac{Bz_j(W)}{\sqrt{mj}} \qquad (\forall i, j \in N).$$

5 Conclusions

Thus we have several conclusions. Consider the issues Q.1 and Q.2 raised in section 2. The mere statements of Q.1 and Q.2 look now confusing in themselves. The reason is the narrowness of the framework in which they were formulated.

First, starting with the Banzhaf index, in the light of the conceptual and formal framework summarized above, it can be said that in a 'take-it-or-leave-it' committee the Banzhaf index seems an adequate measure of decisiveness founded in probabilistic terms, *although in such committees the relevant notion is that of success.* Thus, in the light of the above analysis, the popular square root rule, which gathers ample support in view of providing a priori equal chances of being decisive, appears ill-founded in a double sense: first, it should be the goal of equalizing the likelihood of success what justifies it; and, second and more importantly, *it is only in the context of 'take-it-or-leave-it' committees that this recommendation makes sense.* Most often supporters of this choice apply it to real world committees that develop their decision-making in an environment closer to that of a bargaining committee than to that of a 'take-it-or-leave-it' committee.

Second, in the case of bargaining environments either the Shapley-Shubik index or any other power index, when seen through the glasses provided by formulae (4) or (3), appear as *the 'bargaining power' in the precise game-theoretic sense (i.e. the weights of an asymmetric Nash bargaining solution) that the voting rule gives to each member of the committee, and note that this bargaining power is related to decisiveness. But when the preference profile is TU-like the bargaining power of each player coincides with his/her expected payoff as given by* (5) and (6). Note this dissolves dilemma Q.1. The Shapley-Shubik index appears among all those fitting formula (3) as a remarkable candidate to measure the a priori bargaining power in such committees. In this case the support is not probabilistic but either cooperative-axiomatic or noncooperative game-theoretic (as a limit case).

Finally, there is the question of the quite different recommendations for the choice of voting rule in a committe of representatives depending on its character, either 'take-it-or-leave-it' or bargaining. It seems rather disturbing the different recommendations one obtains from the analysis summarily described above. Even more considering that real-world committees act often in an environment intermediate between the two *pure* types considered here. This does not invalidate these recommendations, on the contrary a clear and sound conceptual founding only sets clear limits to the validity of the conclusions one may get from formal models, while unclear situations underlying models and conceptual vagueness at the base of theory definitely make obscure the sense and validity of any conclusion.

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