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Decision-making Power in Hierarchies

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Abstract This paper introduces a new approach for analysing decision-making power in hierarchies. We show that all existing approaches suffer from the same pathology, i.e. from ignoring that decision-making in hierarchies is sequential. They ascribe power to actors even if those are not allowed to act. Introducing the idea of truncated hierarchies we provide an intuitively plausible method to solve the problem. Formally representing the decision-making process in a hierarchy as an extensive game from, we derive a power score and a power measure and discuss their implications.

Keywords collective decision-making, decision-making power, extensive game form, hierarchies, power

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Extended Abstract This paper introduces a new method for determining the extent of decision-making power in hierarchies. The conventional set-up is characterised by a hierarchically structured group of actors and a collective decision rule that is used to select one of two mutually exclusive alternatives (approval or rejection of a proposal). A collective decision can be made by any sub-group of actors that satisfies the decision rule and obeys the hierarchical relations among the group members. In order to ascribe decision-making power to an actor the existing approaches first identify which subsets of actors can make a binding decision and then, in a subsequent step, count how often each actor is able to alter the outcome of the decision by ceteris paribus changing their own decision. This method is adopted from the well-established method of measuring power in non-hierarchical set-ups. The difference only lies in the reduction of the minimal admissible sub-sets of actors for making a binding decision by taking into account the hierarchical relations. However, the measures based on this idea ignore that decision-making in hierarchies can be expected to be sequential with the result that they ascribe power to actors even if these actors are not allowed to participate in the decision-making given the sequential nature of the decision-making process. In this paper we correct this error by introducing the idea of truncated hierarchies. Formally, we represent the decision-making process in a hierarchy as an extensive game from. Based on this representation we derive a power score and a power measure and discuss their implications.

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> Like Humpty Dumpty, we will insist that a word means what we want it to mean. But if our aim is to construct a body of science, and if we already have in view the general range of phenomena to be explained, our definitions may be wilful, but they must not be arbitrary. If we were to say that we would measure a man's power by his height, this would be an internally consistent definition, but only hardly useful in exploring the phenomena referred to in common speech as the phenomena of power.
>
> — H.A. Simon (1953)

1. Introduction

Power and its analysis in any kind of organisational architecture has received considerable attention in sociology, economics, and political science. The distribution of power is regarded as a fundamental property when it comes to the analysis of any kind of organisational architecture. Daudi (1986: 3) deems it to be a central ingredient in the development of intra-organizational processes. For instance, it has a strong impact on the attitudes and the behaviour of the members of an organisation, and, thus, on its performance (Daudi 1986, Johnston & Gill 1993, Martin 1998) and it is crucial when it comes to ascription of responsibility (Braham & Holler 2005b) which is seen as a vital parameter determining the success of an organisation (Tannenbaum & Kahn 1957, Chandler 1966).

Among all kinds of organisational architectures, hierarchies play not only a major role in the literature, but can also be found in many areas of any society or as Daudi (1986: 8) puts it: 'hierarchies … have existed since the primitive communities of the Stone Age up until the complex industrial societies of today, even if the forms assumed and the degree of extremity have varied extensively'. Typical questions addressed in the literature are whether there exists a tendency of organisational architectures to become hierarchical or how the efficiency of an organisation is related to its architecture (Mackenzie 1976: 98 ff., Radner 1992: 1391 ff.). When it comes to the analysis of power in hierarchies a number of studies concentrate on a specific notion of power known as *power over* or *social power* (Brams 1968, van den Brink 1994, van den Brink & Borm 1994, van den Brink & Gilles 1994, 2000, 2003, Copeland 1951, Daudi 1986, Herings *et al*. 2005, Hu & Shapley 2003, Mizruchi & Potts 1998, Russett 1968, and the references therein).[1] This allows the analysis of authority relations, i.e. the analysis of abilities to enforce obedience. However, when it comes to (collective) decision-making within a hierarchy and the ascription of responsibility for those decisions this notion

[1] For a bibliographical essay devoted to the measurement of the exercise of social power see, for instance, Cartwright (1965).

of power turns out to be inappropriate. What is required is a notion of power which informs us about the individual abilities of its members to force decisions. We call this notion of power *decision-making power* (or *voting power*) which belongs to the more general category of *power to* (or *outcome power*). Even decision-making power and its application has become quite popular during the past two decades, in particular in the context of decision-making in non-hierarchical organizations (Felsenthal & Machover 1998, Holler & Owen 2000, 2001, and 2002, Holler *et al*. 2002, Holler & Gambarelli 2006, and the references therein) to our best knowledge only a handful of studies by van den Brink, Gilles & Owen (van den Brink 1994, 1997, 1999, 2001, Gilles *et al*. 1992, Gilles & Owen 1994, and van den Brink & Gilles 1996), Berg & Paroush (1998), Shapley & Palamara (2000a, 200b), and Steffen (2002) make use of this notion for analysing hierarchies.

As indicated above, usually, studies analysing decision-making power are concerned with decision-making bodies which are non-hierarchical, i.e. *flat*. The canonical set-up can be characterised by a group of *independent* actors, a set of votes assigned to the actors, and a collective decision-making rule for proposals presented to the whole decision-making body. Decision-making is *binary* and *simultaneou*s, i.e. all actors have a free choice between two mutually exclusive alternatives (which can be the approval or rejection of a proposal) and have to choose one of those at the same time. Via the decision-making rule their individual choices are then aggregated into the collective choice of the decision-making body. Such a rule typically requires counting the number of votes for both alternatives and provides a criterion to determine which alternative is collectively chosen. Thus, any sub-group of actors that has enough votes to fulfil this criterion can force a binding collective decision. Note that this set-up can formally also be represented by a *strategic* (or *normal*) *game from*.

If we analyse decision-making power in such a body the occurrence of power differentials can be traced back to the existence of an unequal distribution of votes among the actors. The measurement of decision-making power works essentially as follows: (i) identify the sub-groups of actors which can make a binding collective decision, (ii) ascribe power to individual actors in these subgroups if these actors are, ceteris paribus, able to alter the collective decision, and (iii) aggregate the individual power ascriptions of each actor.

If we want to investigate decision-making power in *hierarchical* decision-making bodies we have to realize that the only things that both have in common are that there is a group of actors and collective decision-making rule which we assume to be binary. Apart from that, in a hierarchy the actors are no longer independent, but are connected via a system of *dominance relations*, and can be characterised by their position in this system. Moreover, decision-making is usually *sequential* which has the effect that in certain cases actors may be excluded from decisions. Thus, here power differences can be traced back to the position of an actor within the hierarchy. While the current approaches for measuring decision-making power in hierarchies, which are all adoptions of well-established methods of measuring power in non-hierarchical set-ups, account for the dominance relations between the actors by reducing the minimal admissible sub-groups of actors for making a decision by obeying the hierarchical relations, they ignore that decision-making can be expected to be sequential. The latter has the effect that those measures ascribe power to actors in sub-groups of actors which by the logic of the hierarchy will never form.

Taking into account the sequential nature of the decision-making rule in a hierarchy, we provide an intuitively plausible method to solve the problems by introducing the idea of *truncated* hierarchies creating an ensemble of truncated hierarchies with overlapping members. Our point of departure is the sub-group of actors which can be confronted with a proposal coming from the outside world on which a decision by the hierarchy is at stake. For each of these actors, we determine which other members of the hierarchy may be involved in the collective decision-making process if this actor would receive a proposal on his desk. Having identified those actors we cut off the other 'irrelevant' actors which are then excluded from the collective decision-making process in this particular case and we obtain a truncated hierarchy. Doing the same for all actors which can receive proposal from the outside world we obtain an ensemble of truncated hierarchies with overlapping members. Note that the exclusion of actors for each of the truncated hierarchies is determined by the outside world, i.e. by the fact where a proposal arrives at the desk of a member of the hierarchy. Furthermore, there is a second type of exclusion which an actor may encounter. This can happen to all actors who have no contact to the outside world, i.e. who are not in a position to obtain a new proposal on their desk. Due to the sequential nature of the decision-making process it can simply happen, that a collective decision is made *before* they have been asked for their opinion. Thus, in this case the exclusion from the collective decision-making is due to the preceding decisions of other actors in the hierarchy. Formally, we can take into account both types of exclusion by representing the decision-making process in hierarchy by an *extensive* (or *sequential*) *game form* including *nature* as an additional actor.

Taking these aspects into account we develop a score and a measure of decision-making power in hierarchies and discuss their implications.

The contributions of this paper can be summarized as follows: we demonstrate that (i) if collective decision-making is rather sequential than simultaneous, existing measures of decision-making power are deemed to fail, and (ii) that power measures based on game forms allow for a much wider range of applications than measures based on simple games. (iii) Based on an extensive game form we develop an approach to analyse power in decision-making bodies which allow decision-making procedures to sequential *or* simultaneous. As every decision-making situation can be represented by a game form, this amounts in the following insight: while a solution concept like the Nash equilibrium helps us to understand the nature of a game, i.e. it tell us something about the payoffs actors can expect, a power analysis of its underlying game form helps us to understand the architecture of the game, i.e.

it informs us about the abilities actors posses to achieve certain outcomes. Furthermore, we show (iv) that existing measures for the analysis of decision-making power in hierarchies implicitly assume decision-making to be simultaneous while it seems rather natural that to be sequential and, (v) that our new approach is appropriate to evaluate decision-making power in hierarchies (even) if decision-making is sequential.

2. Directed Graphs

Informally, a directed graph is a set of objects, called nodes joined by directed links called directed edges. Typically a graph is depicted as a set dotes (the nodes) joined by directed lines (the directed edges). In our paper we will make use directed graphs (i) to describe dominance relations in hierarchies and (ii) to analyse the decision-making power in hierarchies by representing the decision-making process by an extensive game form.

Formally, a *directed graph* (or *digraph*) is an ordered pair $(\mathcal{V},\mathcal{D})$, where $\mathcal{V}$ is a finite set of *nodes* (or *vertices*) of the graph and $\mathcal{D}$ is a set of ordered pairs of $\mathcal{V}$ called directed *edges* (or *arcs*) of the graph, i.e. $\mathcal{D}\subset\mathcal{V}\times\mathcal{V}$ is a binary relation on $\mathcal{V}$. An edge $e =_{def} (i,j)\in\mathcal{D}$ is considered to be directed from *i* to *j* where *i* is called the *predecessor* of *j* and *j* is called the *successor* of *i* in $\mathcal{D}$. By $\boldsymbol{\mathcal{S}}(i)$ and $\boldsymbol{\mathcal{S}}^{-1}(i)$, respectively, we denote the set of all successors and predecessors of $i\in\mathcal{V}$ in $\mathcal{D}$; i.e. $\boldsymbol{\mathcal{S}}(i)=\{j\in\mathcal{V}\,|\,(i,j)\in\mathcal{D}\}$ and $\boldsymbol{\mathcal{S}}^{-1}(i)=\{j\in\mathcal{V}\,|\,(j,i)\in\mathcal{D}\}$.

A node *i* is called a *terminal* node, if no edge is starting from it. If $(i,j)\notin\mathcal{D}$, we shall say that there is a path from a node *i* to *j*, if *i* and *j* are connected via other nodes such that *j* can be reached from *i* by following the edges of the directed graph, e.g. if $(i,h),(h,k),(k,j)\in\mathcal{D}$ then there exists a path from *i* to *j* denoted by $i\to h\to k\to j$ and we call *i* an *ancestor* of *j* and *j* a *descendant* of *i*. Furthermore, let us denote the set of all nodes being descendants of a node *i* by $\hat{\boldsymbol{\mathcal{S}}}(i)$ and the set of all ancestors of *i* by $\hat{\boldsymbol{\mathcal{S}}}^{-1}(i)$.

A directed graph is said to be a *tree* $\mathcal{T}$ if (i) there exists a distinguished node $r\in\mathcal{V}$ (the root of the tree) that has no edges going into it, and (ii) for every other node $i\in\mathcal{V}\setminus\{r\}$ there exists exactly one path from *r* to *i*. We will denote the unique path joining a node *i* to a node *j* (in a tree) by $\mathcal{P}(i,j)$, e.g. if $(i,h),(h,k),(k,j)\in\mathcal{D}$, then $\mathcal{P}(i,j) =_{def} i\to h\to k\to j$.

A *branch* of a tree $\mathcal{T}$ is a directed graph having nodes starting at a node $i\in\mathcal{V}$ and containing all its descendents together with their original edges. We shall denote by $\mathcal{T}_i$ the branch starting at *i*. Thus, $\mathcal{T}_i$ is itself a tree whose root is *i*.

A tree $\mathcal{S}$ is called a *sub-tree* of a tree $\mathcal{T}$, if (i) $\mathcal{V}'\subset\mathcal{V}$ with $\mathcal{V}'$ being the set of nodes of $\mathcal{S}$, (ii) the edges between the nodes of $\mathcal{S}$ are precisely the same edges joining the nodes when considered as nodes of $\mathcal{T}$, (iii) the terminal nodes of $\mathcal{S}$ form a subset of the terminal nodes of $\mathcal{T}$, and (iv) $\mathcal{S}$ and $\mathcal{T}$ have the same root.

3. Hierarchies

Let us define an *organisation* as a group of people interacting in order to achieve certain goals (Senior 2002)[2] and its *architecture* to be a description of the arrangements of its members called *actors*, its decision-making, communication, and information processes (Sah & Stiglitz 1986). Thus, the *organisational architecture* is an abstract shell uninhabited by real actors, i.e. it abstracts from social behaviour as determined by individual preferences.[3] To put it in other words: the architecture can be regarded as the 'hardware' of an organisation, while actors can be seen as the 'software' which is acting within the architecture.

Hierarchies form a certain subclass of organisational architectures. Following van den Brink (1994) they distinguish themselves from other architectures by the arrangement of its members being connected via directed binary relations, which we interpret as *dominance* (or *superior to*) *relations*. Loosely speaking, we can say that an actor in a dominating position has an influence on the 'power' of the other actor who is a position that is dominated by him. Domination can be either *indirect* or *direct*, i.e. with or without intermediate actors, respectively. Actors in dominating positions are called *superiors* (or *principals*) - bosses or managers in common parlance -, while the actors in dominated positions are called *subordinates* (or *agents*). If we refer to a superior who directly dominates another actor, the dominating actor is called a *predecessor* and if we refer to a subordinate who is directly dominated by another actor, the dominated actor is called a *successor*.

Thus, formally, we can represent the dominance structure of a hierarchy as a *directed graph*, where the nodes $j \in \mathcal{V}$ represent the actors $i \in N$, i.e. $\mathcal{V} = N$, and the edges indicate direct dominance relations between the actors, i.e. if there is a directed edge (i, j) we say that predecessor *i* dominates successor *j* and that *j* is dominated *i*. Moreover, we say $\forall\ j \in \hat{\mathcal{S}}(i)$ that *j* is a subordinate of *i*, and $\forall j \in \hat{\mathcal{S}}^{-1}(i)$ that *j* is a superior of *i*. Finally, let us denote the collection of all dominance structures on *N* by $\mathcal{S}_D^N$.

The dominance relations in a hierarchy have the following three properties (see, e.g., Radner 1992 for a similar set of properties):

(i) *Transitivity* – if *i* directly dominates *j*, and *j* directly dominates *k*, that *i* indirectly dominates *k*

(ii) *Anti-symmetry* – if *i* directly dominates *j*, then *j* does not directly dominate *i*; in this case we shall say that *j* is directly dominated by *i* (or *j* is a *successor* of *i*).

[2] For a similar definition see, for instance, Cartwright (1965) who defines an organization as 'an arrangement of interdependent parts, each having a special function with respect to the whole. … [A]n organization has a primary objective (also referred to as purpose, goal, task, function, or mission). To reach this objective, subgoals must be established'.

[3] See also Lévi-Strauss (1963), how uses the term structure instead of architecture

Figure 1 Feasible dominance structures in hierarchies with two and three actors

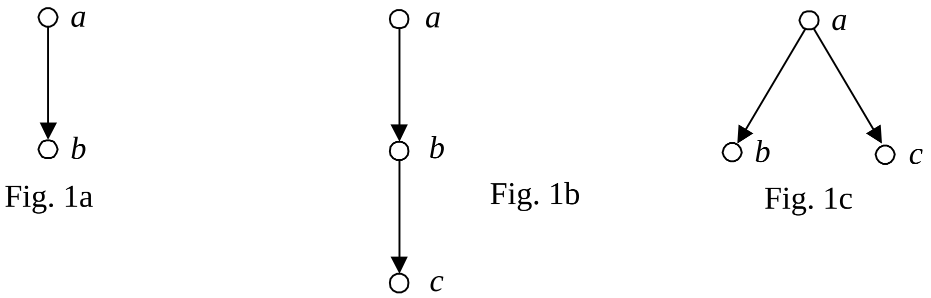


Fig. 1a Fig. 1b Fig. 1c

(iii) *Single topness* – there is exactly one actor, called the *top* (or *root*), who is in a position such that he dominates all other actors. Except for the top, every other actor has at least one predecessor. Formally, there exists an $i \in N$ such that $\hat{\mathcal{S}}(i) = N \setminus \{i\}$ and, thus, $\hat{\mathcal{S}}^{-1}(i) = \varnothing$ (van den Brink 2001 calls this property *quasi-strongly connectedness*).

Thus, a hierarchy is formally a *kind of* tree structure characterised by a collection of actors together with dominance relations among them. Note, that by (i) in combination with (ii) we assume $\forall i \in N : i \notin \hat{\mathcal{S}}(i)$, i.e. we exclude the case that a dominance structure is cyclic. All feasible dominance structures of hierarchies with two and three members which fulfil properties (i) – (iii) are illustrated in Figure 1 and are formally given by $\mathcal{S}(a) = \{b\}$ and $\mathcal{S}(b) = \varnothing$ for Fig. 1a, $\mathcal{S}(a) = \{b\}$, $\mathcal{S}(b) = \{c\}$ and $\mathcal{S}(c) = \varnothing$ for Fig. 1b, and $\mathcal{S}(a) = \{b, c\}$ and $\mathcal{S}(b) = \mathcal{S}(c) = \varnothing$ for Fig. 1c.

Note, that not all actors in a hierarchy are necessarily comparable as a tree is only a partially ordered set, but not necessarily a completely ordered one. For instance, in Fig. 1c, *b* is not dominating *c*, nor is *c* dominating *b* (Radner 1992: 1391). In everyday language, the word hierarchy not only connotes an upside-down-tree-like structure, but also an assignment of rank or level. By a ranking of tree we shall mean an assignment of a number called the *rank* (or *level*) to each actor, such that:

(iv) if *i* dominates *j*, than *i* has a higher rank (larger number) than *j*

(v) if *i* and *j* have the same rank, then they are not comparable, i.e., *i* does not dominate *j*, nor *j* dominates *i*.

Moreover, for the purpose of our analysis it is sufficient to restrict ourselves to dominance structures of the following type:[4]

(vi) if *i* dominates *j* and *i* has rank *k*, than j has rank *k*-1

[4] Note, that otherwise, there may be more than one way to rank a hierarchy (in a way which satisfies properties (iv) and (v)). For instance, the dominance structure $\mathcal{S}(a) = \{b, c\}$, $\mathcal{S}(c) = \{d\}$, and $\mathcal{S}(b) = \mathcal{S}(d) = \varnothing$ fulfils (i) to (iii), but allows *b* to have the same rank as *c* who is also directly dominated by *a* or, alternatively, to have the same rank as *d* who is directly dominated by *c* (Radner 1992: 1391).

Thus, given (iii) the highest rank consists of the top only. Then the second highest rank is formed by the actors, which are directly dominated by the top. In turn, the third level is formed by those actors, which are directly dominated by the actors on the second highest rank and so on. An actor is called to be at the *bottom* (or a *leaf*) if he is not dominating any other position, i.e. if he owns a terminal node. Note, that this does not imply that such a position necessarily belongs to the lowest rank for which we shall adopt the convention that this is 1.

Roughly speaking, the dominance structure of a hierarchy tells us something about the actors' abilities to command others. However, this information is of minor importance if want to analyse the collective decision-making, communication, or information processes in a hierarchy. Even they can be described by directed edges as well it would be mistaken to assume that the dominance relations being are necessarily determining the direction of the collective decision-making, communication, or information processes. In the course of our analysis we will, for instance, show that for the decision-making processes applied in the literature not only the edges have an opposite direction, but also that the arrangement of the actors might change.

In this paper we focus on the architecture of the collective decision-making processes in hierarchies only, which despite its importance for an organisation has surprisingly found little attention yet.[5] So see why collective decisions are of significant importance for an organization it is helpful to classify the types of decisions made in an organisation. Decisions in any organisation of significant size and complexity can be distinguished into so-called *key* and *routine* decisions (see Braham & Steffen 2002 and implicitly Shubik 1962). Routine decisions denote decisions that follow schematic rules and can be delegated to individuals, such as the decision to order a product that is out of stock. By contrast, key decisions refer to decisions that affect the reputation, profitability, or destiny of an organization and contain a large element of risk and uncertainty'. In the context of a firm we can think of the decision on a major investment, the liquidation of a department or subsidiary, takeovers or mergers, and so forth. In contrast to routine decision key decisions are usually an outcome of a series of individual decisions (within the hierarchy), i.e. a collective decision-making process.

With regard to the nature of the decision-making rule (DMR) τ applied to a collective decision making process, we can derive six types of DMRs from the literature. They differ in (i) the set of actors that has contact to the outer world and can receive new proposals on their desk, (ii) the set of predecessors that may be involved in a collective decision about a proposal presented to an actor who has a contact to the outer world, and (iii) the set of

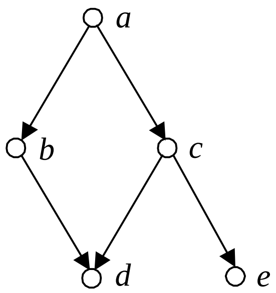
[5] Note that there exists an own well-developed field in the literature which deals with the analysis of these communication and information processes in organisational architectures which are not necessarily to hierarchies. See, for instance, Borm *et al*. (1992, 1994), Keren & Levhari (1983), Koh (1991), Mizruchi & Potts (1998), Myerson (1977), Owen (1986), Rosenthal (1988), Sah & Stiglitz (1985, 1986), and Slikker *et al*. (2005) for the analysis of communication processes, Radner (1992) for the analysis of information processes, and Chwe (1995, 2000), Orbay (2001), and Bolton & Dewatripont (1994) for studies of both.

predecessors required for an approval of such a proposal. All six rules are illustrated by Table 1.

Table 1 Collective DMRs in hierarchies[6]

τ	Actors *i* with a contact to the outer world	Predecessors of *i* potentially involved	Predecessors of *i* required for an approval
1	All	All	Top
2	All	All	All on one path up to the top
3	All	All	All
4	Bottom positions only	All	Top
5	Bottom positions only	All	All on one path up to the top
6	Bottom positions only	All	All

Figure 2 A dominance structures with five members



To illustrate the difference of the rules by an example we have to make use of the dominance structure shown in Fig. 2 and formally given by $\mathcal{S}(a)=\{b,c\}$, $\mathcal{S}(c)=\{d,e\}$, $\mathcal{S}(b)=\{d\}$, and $\mathcal{S}(d)=\mathcal{S}(e)=\varnothing$.

[6] See, for instance, van den Brink 1994, 1997, 1999, 2001, Gilles et al. 1992, Gilles & Owen 1994, and van den Brink & Gilles 1996, Sah & Stiglitz 1985, 1986, 1988, Ben-Yashar & Nitzan 1998, 2001 and Koh 1991, 2005, 2006, and Shapley & Palamara 2000a, 2000b.

Table 2 Collective DMRs applied to Fig. 2

τ	Actors *i* with a contact to the outer world	Superiors of *i* potentially involved	Superiors of *i* required for an approval
1	a		
	b	a	a
	c	a	a
	d	a,b,c	a
	e	a,c	a
2	a		
	b	a	a
	c	a	a
	d	a,b,c	$(a \wedge b) \vee (a \wedge c)$
	e	a,c	$(a \wedge c)$
3	a		
	b	a	a
	c	a	a
	d	a,b,c	$(a \wedge b \wedge c)$
	e	a,c	$(a \wedge c)$
4	d	a,b,c	a
	e	a,c	a
5	d	a,b,c	$(a \wedge b) \vee (a \wedge c)$
	e	a,c	$(a \wedge c)$
6	d	a,b,c	$(a \wedge b \wedge c)$
	e	a,c	$(a \wedge c)$

Thus, roughly speaking, a DMR in a hierarchy describes how an outcome can be produced by defining which subsets of its members are able to do this.

4. Power

Our basic understanding of 'power' follows Harré (1970), Wrong (1995), Morriss (1987/2002) and Braham & Holler (2005a) who define power as a concept that always refers to a generic (and therefore, in a sense, timeless) ability or capacity of an object. Ordinarily speaking, a power ascription refers to an object's ability: what an object is *able* to do. Let us consider the subclass of objects called *actors*. If an actor *i* wanted a particular outcome and has an action (or sequence of actions) such that its performance under stated or implied conditions will result in that outcome and would not result of *i* would not perform this action (or sequence of actions), we ascribe power to such an actor under the specified conditions, i.e. we claim that *i* is *able to do* under specified conditions irrespective of their occurrence. The latter refers to the dispositional property of power being a *capacity* or *potential* which exists whether it is exercised or not. In current terminology, power as a generic ability is what is called *power to* or *outcome power* (Braham & Holler 2005a: 145).[7]

In our context, the ability in question is the ability of actors is to effect (i.e. to 'force' or 'determine') outcomes as a result of a collective decision in a hierarchy. That is, an actor has an action, which, if chosen, will make a decisive difference to the outcome. Thus, we deal with a subset of 'powers to' which we call *decision-making power*. The measurement of decision-making power involves three to four steps: (i) the identification of the sub-groups of actors within the decision-making body which can make a collective decision, (ii) the power ascription to individual actors in these subgroups: we ascribe power to an actor, if the actor is able to alter the outcome of the collective decision by *ceteris paribus* changing his own decision, i.e. a power ascription does not say how much power an actor has, only that there exist circumstances in which he is non-redundant for the outcome, (iii) the aggregation of the individual power ascriptions of each actor, and, if we want more than what is known as a bare power *score* (or *ranking*), (iv) a weighting of the aggregated power ascriptions. If this weighting is such that all aggregated power ascriptions sum up to unity we say that we have a power *index*, otherwise we talk about a power *measure*. While a score just allows us ordinal comparisons of power structures among decision-making bodies, a measure makes cardinal comparisons feasible.

Finally, note that as we are concerned with a structural (or constitutional) analysis the type of power we are considering what Morriss (1987, 2002: 107 f.) calls the 'power of a position' in contrast to the 'power of a named individual', i.e. the power which we analyse is based on positions (and the set of rules) in an organisational architecture and is independent from the powers of the individual actors which are placed in those positions. Thus, our power ascriptions refer to abilities attached to positions of an architecture: what an actor placed in a position is able to do by the characteristics of such a position and its relations to others within the architecture.

[7] Another basic notion of power is the notion of *power over* or *social power*. The distinction between both notions which was originally made by Oppenheim (1961, 1981), is discussed in Morriss (1987/2002: 32-5), Dowding (1991: 48-51, 1996: 1-18) and Wrong (1995: xix-xxv). The difference is essentially that *power to* concerns an actor's 'ability to bring about or help to bring about outcomes' (generic ability), while *power over* concerns 'the ability of an actor to change the incentive structure of another actor or other actors to bring about, or help bring about outcomes' (Dowding 1991: 48) (i.e. *power over* is an asymmetric relation between two or more actors). Note that it should be obvious that *power over* implies *power to*: *i* cannot extract any concession from *j* (has *power over*) unless *i* has the *power to* do, something to *j* that *j* cannot do to *i* (Braham & Holler 2005a: 152).

5. Analysing Power in Non-Hierarchical Decision-making Bodies

As already mentioned in Section 1 all approaches for measuring decision-making power in hierarchies are all adoptions of well-established methods for measuring decision-making power in non-hierarchical set-ups. We can find adoptions of classical measures such as the Shapley & Shubik (1954) index and the Banzhaf (1965) measure. In order to understand the proposed adoptions of these measures for decision-making power in hierarchies, it seems to be appropriate to begin with a brief introduction into the original measures for non-hierarchical decision-making bodies.

As indicated above, the canonical set-up for measuring decision-making power in flat decision-making bodies can be characterised by a group of independent actors, a vector of weights containing the votes assigned to each actor, and a binary and simultaneous collective DMR for proposals presented to the whole decision-making body. As an illustration we can think of a group of actors sitting around a table who have to decide on proposal being presented to the whole group making their choice by pressing a decision button at the same time. The only difference between those actors may be that their individual decisions have different weights in the collective decision.

Formally, this set-up is usually represented by a *simple game* which is a pair $(N, \mathcal{W})$ where $\mathcal{W}$ is a collection of subsets (coalitions) of the finite *set of actors* $N =_{def} \{1,...,n\}$, satisfying the following three conditions: $\varnothing \notin \mathcal{W}$; $N \in \mathcal{W}$; and (monotonicity) if $S \in \mathcal{W}$ and $S \subseteq T$, then $T \in \mathcal{W}$. A subset S is said to be *winning* or *losing* according to whether $S \in \mathcal{W}$ or $S \notin \mathcal{W}$. Furthermore, a subset S is called a *minimal winning coalition* iff $S \in \mathcal{W}$, but no subset of S is in $\mathcal{W}$. The set of minimal winning coalitions is denoted by $\mathcal{M}$.

Thus, the DMR can be represented by $\mathcal{W}$ as it specifies which subsets of actors can ensure the acceptance of a proposal or by $\mathcal{M}$ which specifies which subsets of actors can minimally ensure this. This can be illustrated by a *weighted voting game* $[q; w_1, w_2, \ldots, w_n]$ where w_i represents actor i's (non-negative) *voting weight* and q with $0 < q \leq \sum(i \in N : w_i)$ is the *quota* of votes necessary to establish a winning coalition, i.e. to accept a proposal.[8]

The canonical decision making situation can also be represented by a *strategic* (or *normal*) *game form* (SGF) which is a deterministic mapping of action *n*-tuples into outcomes.[9] A SGF is generally defined by (i) N such that each actor $i \in N$ has an *action set* $A_i =_{def} \{a_{i1},...,a_{iq}\}$ with a_{ik} being an action of actor i (which we assume to be unconditional) and (ii) an *outcome function* $f : A_1 \times ... \times A_n \rightarrow \mathbb{O}$ that assigns to each *action profile* $\alpha_h =_{def} (a_1,...,a_n)$ being an element of the *set of actions profiles* $\mathcal{A}$, i.e. the set of all possible action combinations of the

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[8] Note, that even each simple game can be represented as a weighted voting game it is not necessarily hold that it is a weighted voting game as it might not be weighted (see Taylor and Zwicker 1999 for further details).

[9] The idea of a game form has been introduced by Gibbard (1973) to describe games in which individual utilities are not yet attached to possible outcomes (in effect a game without payoff functions). For an introduction into strategic (or normal) game forms and its application to the measurement of power see Miller (1982).

involved actors, an outcome o belonging to the *outcome set* $\mathbb{O}$.[10] Thus, a SGF is sufficiently represented by its outcome function *f*. In our case, i.e. for the canonical set-up for measuring decision-making power in flat decision-making bodies, we have $\forall i \in N : A_i = \{yes, no\}$, $\mathbb{O} = \{acceptance, rejection\}$ and f ,like $\mathcal{W}$, taking into account the decision-making rule and the weights of the actors, i.e. determining which action profiles ensure an acceptance or a rejection of a proposal. Thus, a coalition *S* in a simple game approach can be regarded as an 'index' of the actions of the actors which have chosen the same action, for instance, '*yes*', if $S \in \mathcal{W}$ (Braham 2006).

Note, that both representations can be applied interchangeably if we are concerned with the analysis of decision-making power in flat decision-making bodies. Even the former is the one which is conventionally used we will see that there exists no natural analogue properly to represent decision-making in a hierarchies, while this exists for the latter. Therefore, where possible, we will make use of the latter representation to make it easier for the reader to compare the analysis of power in flat and hierarchical decision-making bodies.

For ascribing power to an actor $i \in N$ under the latter representation we start with a combination of step (i) and (ii) examining for each action profile a_h whether *i* can alter the resulting outcome o by changing his action leaving the actions of all other actors constant, i.e. under a c.p. condition. If *i* is able to do so, i.e. if *i* can alter the outcome, we say that *i* has a *swing* (or is *critical*) and we ascribe power to *i*. We say that *i* has a *positive swing*, if *i* by switching from a '*no*'- to a '*yes*'-action can alter the outcome from a '*rejection*' to an '*acceptance*' and we say that *i* has a *negative swing*, if *i* by switching from a '*yes*'- to a '*no*'-action can alter the outcome from an '*acceptance*' to a '*rejection*'.[11]

Under step (iii) the individual power ascriptions are aggregated by adding up the swings of each actor. This leads to what is known as a power score (or ranking) (see, e.g., Felsenthal & Machover 1998) η_i. Formally, it is given by:

(1) $$\eta_i(f) =_{def} \sum \{a_h \in \mathcal{A} \mid i \text{ is a swinger in } a_h : 1\}$$

If we continue with step (iv) in order to obtain a power measure, we apply different weightings to actions profiles and, thus, to the related power ascriptions, i.e. to the swings of the actors. The two most known measures are the Banzhaf (1965) and the Shapley-Shubik (1954) measures. While the Banzhaf measure β' (formally) assigns equal weight to each

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[10] Note, that the literature on power sometimes uses the term 'strategy' as a synonym for an 'action' (see, e.g., Miller 1982 and more recently Braham and Steffen 2003 or Braham and Holler 2005a) even decision-making power is defined as the ability of an actor to effect outcomes by his chosen actions and not by his plan of action, i.e. his strategy being a complete contingent plan, that specifies how an actor will act in every possible distinguishable circumstance in which he might be called upon to move (Mas-Colell *et al*. 1995: 228). However, both coincide if an actor is free to perform any strategy he has chosen, i.e. if other actors cannot prevent him from performing it (by exercising their strategies) (Basar & Olsder 1982). Put it in other words: both coincide if an individual decision can be made independent from others. While this is the case for the canonical set-up of a flat decision-making body, it does not necessarily hold any longer for a hierarchical body. Therefore, we will refer to actions only and not to strategies.

[11] Thus, *swings* (see section 5) are *power to* and do not entail *power over*, taken as the 'ability to extract concessions'.

action profile (of which i is a member and, thus, could be a swinger), the Shapley-Shubik index ϕ assigns to each action profile the weight $x!(n-x)!/(n+1)!$ where x denotes the number of actors $i \in N$ in an action profile a_h who have decided for a '*yes*'-action (Felsenthal & Machover 1988: 207 ff.). Formally, the measures are given by:

(2) $\beta'_i(f) =_{def} \sum\{a_h \in \mathcal{A} \mid i \text{ is a swinger in } a_h : 1/\#\mathcal{A}\}$

(3) $\phi_i(f) =_{def} \sum\{a_h \in \mathcal{A} \mid i \text{ is a swinger in } a_h : x!(\#N-x)!/(\#N+1)\}$

In order to illustrate how these measures are functioning and that an unequal distribution of weights is a necessary condition for power differentials let consider the following three examples:

Example 1 Assume $[2;1,1,1]$. Here four action profiles lead to an acceptance of a proposal: (*yes*, *yes*, *no*), (*yes*, *no*, *yes*), (*no*, *yes*, *yes*), (*yes*, *yes*, *yes*), while the remaining four profiles lead to a rejection. From this we obtain all actors have each two positive and two negative swings. This yields to $\eta\ (f) = (4,4,4)$, $\beta'(f) = (0.50, 0.50, 0.50)$ and $\phi(f) = (0.33, 0.33, 0.33)$

Example 2 Assume $[3;2,2,1]$. This example leads to the same results as Example 1, i.e. we have the same four action profiles which lead to an acceptance of a proposal and, thus, we obtain $\eta\ (f) = (4,4,4)$, $\beta'(f) = (0.50, 0.50, 0.50)$ and $\phi(f) = (0.33, 0.33, 0.33)$

Example 3 Assume $[4;3,2,1]$ In this case only three action profiles lead to an acceptance a proposal: (*yes*, *yes*, *no*), (*yes*, *no*, *yes*), (*yes*, *yes*, *yes*). From this we obtain that *a* has three positive and three negative swings, while *b* and *c* have each one positive and one negative swing. This yields to $\eta\ (f) = (6,2,2)$, $\beta'(f) = (0.75, 0.25, 0.25)$ and $\phi(f) = (0.67, 0.17, 0.17)$

6. Analysing Power in Hierarchical Decision-making Bodies: Existing Approaches

Although the analysis of decision-making power in flat decision-making bodies has become increasingly popular in the second half of the last century and the use of tools of game theory for analysing hierarchical bodies was foreseen many years ago by Morgenstern (1951), Shapley (1962, p. 66), and Shubik (1962), the subject has been more or less left by the wayside.[12] Early notable exceptions are the contributions by van den Brink (1994, 1997, 1999, 2001), Gilles *et al*. (1992), Gilles & Owen (1994), and van den Brink & Gilles (1996). In order to take into account the aspects of decision-making power in hierarchies they

[12] Note, that there exists an in some way related field of research in sociology (see, for instance, Tannenbaum & Kahn 1957, Tannenbaum 1961, and Smith & Tannenbaum 1963) concerned with the *exercise* of *control* of actors over the collective decisions in hierarchical organisations, but not with *bare ability* to control. As control is here defined as the effect an actor has in determining the actions of an organization it appears to be either a sub-category of what we call decision-making power (see Tannenbaum & Kahn 1957) or more or less the same (see Tannenbaum 1961). However, given that these scholars are concerned with the exercise of it they make use of a quite different methodology, i.e. a set of questions to be answered by the members of hierarchy.

represent the decision-making situation as a combination of a simple game which represents a decision rule for a flat decision-making body and a so-called *permission structure* which takes into account the dominance relations and a *permission rule* for restricting the minimal admissible sub-groups of actors for making a collective decision, i.e. $\mathcal{M}$ in the language of simple games or applying the representation of a SGF: the subset of all action profiles for which every actor opting in favour of a proposal has a negative swing. A permission rule is a rule which gives the set(s) of superiors that can approve the decision of an actor *i* (see column "Superiors required for an approval" in Table 2). Based on the combination the DMR for a flat body and the permission structure they come out with a Shapley and a Banzhaf *permission value* which can be interpreted as measures of decision-making power if applied to simple games (see Gilles *et al*., 1992).[13]

More recently, also Shapley & Palamara (2000a, 2000b) have developed a simple game theoretic framework for measuring decision-making power in hierarchies which they equate with the notion of responsibility. For the derivation of their power measure they introduce four types of games. They begin with the introduction of command game. This game is defined for an actor *i* and informs us about the actors he dominates. Thus, the set of all command games within a hierarchy describes its local authority structure. In order to transfer this information into a global one, they define a control game for each actor which informs us about the coalitions $S \subseteq N$ by which *i* is controlled, i.e. which have power over *i*. Next, they introduce the idea of task game which ignoring the dominance structure of the hierarchy gives us the subsets of actors which have the ability to fulfil a certain task, i.e. can effect a collective outcome. Finally, they compose the two afore mentioned games and call the compound a task control game which is defined for a certain task. For this task it tells us which coalitions in a hierarchy can effect an outcome by their collective decision. Making use of the task control game, which itself is also a simple game, Shapley & Palamara suggest that decision-making power (and responsibility) in a hierarchy can be measured by calculating the Shapley-Shubik index for this game. However, they do not insist that this necessarily the most appropriate measure as they mention the Banzhaf measure is a serious alternative candidate which they have not yet examined.

A third approach can be find in Steffen (2002) who suggests the use of Straffin's partial (1977, 1978) homogeneity approach for measuring decision-making power in hierarchies. This approach which is also based on simple games allows to partition set actors in sub-groups of homogeneous actors. It (formally) contains the Banzhaf measure and the Shapely-Shubik index as extreme cases. We arrive at the Banhaf measure, if we assume that each actor forms his own partition, and at the Shapely-Shubik index, if we assume that there is only one partition including all actors. By its nature this approach allows to take into account the fact

[13] Note, that the central aim of their analysis which is based on TU-games is the creation of allocation mechanisms for the distribution of value within hierarchies, e.g. for payment schemes for employees of a firm.

that actors who participate in a decision-making in general have a *damatis personae*: they are the bearers of predetermined attributes and modes of behaviour. That is, actors in hierarchies often play predetermined *roles*, such as salesman, financial officer, head of external affairs, etc., which are equipped with a bundle of incentive structures. Therefore, it is argued that determining the power of an actor in such a structure must take such information into account, for instance, by partitioning the set of actors into a priori homogeneous subsets, because it is likely that actors with same incentive structure will act in the same way.[14] In order to obey the dominance structure Steffen applies an idea similar to Shapley & Palamara and represents the decision-making process as a compound game. Moreover, he shows that this approach, if all actors have the same incentive structure being either all in one partition or all in an individual partition, is equivalent to van den Brink, Gilles and Owen's measures, i.e. the approach coincides with the Shapley and the Banzhaf *permission value*, respectively.

Finally, we have to refer to Berg & Paroush (1998), whose main concern is not the analysis of decision-making power, but is the analysis of the properties of judgement aggregation in hierarchies. In contrast to the three former approaches their study to a certain degree ignores the existence of the dominance structure as only the ranking of the actors are taken into account. For each rank they define a majority game and compose those into a product game, i.e. a game where each component has to be won in order to win the whole game. Using this compound game they analyse the probability of a correct collective decision of the hierarchy assuming the existence of an exogenous truth. In the course of the derivation of this probability they obtain a binomial probability expression which they call a 'β-number' which coincides with the Banzhaf measure for their compound game. Having stated this without a proof they point out that, thus, their approach can also be applied to analyse decision-making power in hierarchies.

All existing adopted measures which have been developed for measuring decision-making power in hierarchies[15] have in common that they account for the fact that in a hierarchy the actors are no longer independent, but are connected via a system of *dominance relations*, by reducing $\mathcal{M}$, i.e. the number of minimal admissible sub-groups of actors for making a collective decision, by obeying the hierarchical relations. What these measures are essentially doing – partly by introducing an impressive additional formal framework – is that they make use of the formulas above applying the collective DMRs τ = 1, …, 6 introduced in Table 2 for pinning down $\mathcal{M}$. To put it in other words as discussed in Steffen (2002): they take a DMR in a hierarchy to be a simple game where the DMR is designed in a way that hierarchical relations are taken into account. Therefore, we think it is legitimate to omit the formal introduction of the adopted measures and to refer the interested reader to the literature.

[14] Note, that we use the term 'act in the same way' in order to point out that we are not talking about 'cooperation' of players in the sense of Aumann and Drèze (1974).

[15] Note, that this excludes the approach by Berg & Paroush (1998).

However, to illustrate how the above adoptions of the classical measures work we give four examples.[16]

Example 4 Assume $N=\{a,b\}$ and $\boldsymbol{\mathcal{S}}\in\boldsymbol{\mathcal{S}}_D^N$ given by $\boldsymbol{\mathcal{S}}(a)=\{b\}$ and $\boldsymbol{\mathcal{S}}(b)=\varnothing$.

Table 3 Decision-making Power in Fig. 1a

τ	1 – 3	4 - 6
$\mathcal{W}$	$\{a,ab\}$	$\{ab\}$
$\eta(f)$	$(4,0)$	$(2,2)$
$\beta'(f)$	$(1.00,0.00)$	$(0.50,0.50)$
$\phi(f)$	$(1.00,0.00)$	$(0.50,0.50)$

Example 5 Assume $N=\{a,b,c\}$ and $\boldsymbol{\mathcal{S}}\in\boldsymbol{\mathcal{S}}_D^N$ given by $\boldsymbol{\mathcal{S}}(a)=\{b\}$, $\boldsymbol{\mathcal{S}}(b)=\{c\}$ and $\boldsymbol{\mathcal{S}}(c)=\varnothing$.

Table 4 Decision-making Power in Fig. 1b

τ	1	2 – 3	4	5 - 6
$\mathcal{W}$	$\{a,ab,ac,abc\}$	$\{a,ab,abc\}$	$\{ac,abc\}$	$\{abc\}$
$\eta(f)$	$(8,0,0)$	$(6,0,0)$	$(4,0,4)$	$(2,2,2)$
$\beta'(f)$	$(1.00,0.00,0.00)$	$(0.75,0.00,0.00)$	$(0.50,0.00,0.50)$	$(0.25,0.25,0.25)$
$\phi(f)$	$(1.00,0.00,0.00)$	$(1.00,0.00,0.00)$	$(0.50,0.00,0.50)$	$(0.33,0.33,0.33)$

Example 6 Assume $N=\{a,b,c\}$ and $\boldsymbol{\mathcal{S}}\in\boldsymbol{\mathcal{S}}_D^N$ given by $\boldsymbol{\mathcal{S}}(a)=\{b,c\}$ and $\boldsymbol{\mathcal{S}}(b)=\boldsymbol{\mathcal{S}}(c)=\varnothing$.

Table 5 Decision-making Power in Fig. 1c

τ	1 – 3	4 – 6
$\mathcal{W}$	$\{a,ab,ac,abc\}$	$\{ab,ac,abc\}$
$\eta(f)$	$(8,0,0)$	$(6,2,2)$
$\beta'(f)$	$(1.00,0.00,0.00)$	$(0.75,0.25,0.25)$
$\phi(f)$	$(1.00,0.00,0.00)$	$(0.67,0.17,0.17)$

To see that the existing measures are based on nothing more than a simple game that takes into account the hierarchical relations, let us compare examples 1, 3 and 6 with a focus on τ = 4 to τ = 6 for the latter. Examples 1 and 6 share the property that all actors have equal weights and they only differ with respect to the existence of dominance relations among the actors.

[16] Note that our analysis will not consider the case of differing incentive structures between the actors in order to keep the content of the paper traceable which respect to differences in the measurement of power in hierarchies and flat decision-making bodies.

Example 7 Assume $N=\{a,b,c,d,e\}$ and $\mathcal{S}\in\mathcal{S}_D^N$ given by $\mathcal{S}(a)=\{b,c\}$, $\mathcal{S}(c)=\{d,e\}$, $\mathcal{S}(b)=\{d\}$, and $\mathcal{S}(d)=\mathcal{S}(e)=\varnothing$.

Table 6 Decision-making Power in Fig. 2

τ	1	2	3
$\mathcal{W}$	$\{a,ab,ac,ad,ae,abc,abd,abe,acd,ace,ade,abcd,abce,abde,acde,abcde\}$	$\{a,ab,ac,abc,abd,acd,ace,abcd,abce,acde,abcde\}$	$\{a,ab,ac,abc,ace,abcd,abce,abcde\}$
$\eta(f)$	$(32,0,0,0,0)$	$(22,0,0,0,0)$	$(16,0,0,0,0)$
$\beta'(f)$	$(1.00,0.00,0.00,0.00,0.00)$	$(0.69,0.00,0.00,0.00,0.00)$	$(0.50,0.00,0.00,0.00,0.00)$
$\phi(f)$	$(1.00,0.00,0.00,0.00,0.00)$	$(1.00,0.00,0.00,0.00,0.00)$	$(1.00,0.00,0.00,0.00,0.00)$

τ	4	5	6
$\mathcal{W}$	$\{ad,ae,abd,abe,acd,ace,ade,abcd,abce,abde,acde,abcde\}$	$\{abd,acd,ace,abcd,abce,abde,acde,abcde\}$	$\{ace,abcd,acbe,acde,abcde\}$
$\eta(f)$	$(24,0,0,8,8)$	$(16,4,8,8,4)$	$(10,2,10,2,6)$
$\beta'(f)$	$(0.75,0.00,0.00,0.25,0.25)$	$(0.50,0.13,0.25,0.25,0.13)$	$(0.31,0.06,0.31,0.06,0.19)$
$\phi(f)$	$(0.67,0.00,0.00,0.17,0.17)$	$(0.50,0.08,0.17,0.17,0.08)$	$(0.38,0.05,0.38,0.05,0.13)$

Due to the existence of these relations in Example 6 the only dominating actor *a* gains power, while the power of dominated actors *b* and *c* decreases as $\{b,c\}$ no longer belongs to $\mathcal{M}$ (and, therefore, also no longer to $\mathcal{W}$), or to put it in the language of SGFs: the action profile (*no*, *yes*, *yes*) no longer leads to an acceptance of a proposal. However, we obtain the same result, if we assign weights to the actors in Example 1 such as done in Example 3. Comparing Example 3 with Example 6 under $\tau = 4$ to $\tau = 6$ we can notice that $\mathcal{M}$ (and, thus, $\mathcal{W}$) and, consequently, the resulting power distributions are identical. Thus, the hierarchical decision-making situation given by Example 6 under $\tau = 4$ to $\tau = 6$ can be represented as a plain weighted voting game. Similarly, we can find a weighted voting game, which represents the hierarchical decision-making situation in examples 4, 5 and 7.[17]

Let us wind up this section with some observations about the power distributions in our examples. From examples 4 to 7 it becomes obvious that under $\tau = 1, 2, 3$ the top of a hierarchy is all-powerful, i.e. the top is a dictator. However, moving from $\tau = 1$ to $\tau = 3$ the Banzhaf power of the top tends to decline. Moreover, moving from $\tau = 4$ to $\tau = 6$ the power distribution has the propensity to become more equal. Finally, as we can see from Example 7, decision-making power for all three measures is neither necessarily monotone in the rank of actors nor in the number of subordinates if $\tau = 4, 5, 6$ are applied.

7. The Pathologies of Existing Approaches

All existing approaches for the measurement of decision-making power in hierarchies are rooted in a set of assumptions, which the literature has not fully disclosed yet. For instance, it has been clarified that decision-making is assumed to be binary, i.e. that abstention or a simultaneous decisions among more than two alternatives are excluded, and in most cases pointed out that proposals are exogenous (rather than endogenous). However, nothing explicitly has been said about the procedure of the decision-making process, i.e. whether it is simultaneous or sequential. Still one might encounter that this is not really necessary as the existing approaches are based on simple games which are inherently simultaneous, i.e. that the simultaneous nature is an obvious fact that hardly requires spelling out. While we do not object the obvious simultaneous nature, but we show that this assumption is inappropriate when we deal with decision-making

[17] Steffen (2002) has shown that this holds in general.

procedures in hierarchies; something which has remained unnoticed by not making this assumption explicit.

The very basic motivation for the creation of a hierarchy is the idea of delegation, for instance, as a cost reduction device. Assuming decision-making hierarchies to be simultaneous, i.e. assuming a decision-making process that always includes all members, would thwart this idea, and, hence, might be relevant in a few exceptional cases only. Instead, a sequential decision-making process naturally corresponds to the motivation for the creation of a hierarchy as it has the effect that proposals are not necessarily presented to *all* members of the decision-making body, i.e. in certain cases actors may be excluded from decisions - a view that is also in line with opinion in the related research field of judgment aggregation in hierarchies which regards the sequential nature as a recognized and undisputed fact (see, for instance, Sah & Stiglitz 1986 and Koh 1991, 1994, 2005, 2006).

Thus, a measure of decision-making power in hierarchies should explicitly take into account the sequential nature of the decision-making procedure. If it fails to do so, which holds for all existing measures, we can observe power ascriptions to actors in sub-groups, which by the logic of the sequential nature of the decision-making process will never form, or to put it differently, power is ascribed to actors in actions profiles which do not exist. This can have the effect that also the resulting power distributions are faulty.[18]

Consider again Example 6. For $\tau = 4, 5, 6$ applying the existing measures we obtained $\mathcal{W} = \{ab, ac, abc\}$, or the action profiles (*yes*, *yes*, *no*), (*yes*, *no*, *yes*) and (*yes*, *yes*, *yes*) resulting in the acceptance of a proposal, respectively. But why should, for instance, $\{abc\}$ form at all? According to the DMRs considered only bottom-actors have contact to the outer-world, i.e. only these actors can receive new proposal on their desk. Thus, if, for instance, *b* receives a proposal on his desk, he will make up his mind about it. If he decides to support it, *b* requires the approval his predecessor *a*. If *a* shares his view, the proposal is approved, i.e. we have $\{ab\} \in \mathcal{W}$. The same holds if the proposal is presented to *c*, i.e. $\{ac\} \in \mathcal{W}$. But why should it happen that all three approve a proposal, i.e. $\{abc\} \in \mathcal{W}$? Under a flat set-up this can be justified because all actors are sitting around a table and cast their votes simultaneously. Thus, it can happen, that all three actors support

[18] The only paper we are aware of that implicitly points into this direction is Faigle & Kern (1992). Discussing the *Shapely* (1953) *value* for cooperative games under precedence constraints they draw attention to the fact that for Gilles *et al.*'s (1992) *permission value* only the presence of actors in a coalition counts, but not the order by which they have joined the coalition. To put it in other words: the fact that *a* dominates *b* in Fig. 1 a implies that $\{b\} \notin \mathcal{W}$ and $\{a,b\} \in \mathcal{W}$, i.e. *b* 'only "counts" in a coalition if [*a*] is also present', while it is not taken into account 'how a coalition is to be formed',

a proposal even this is not a necessary requirement of the applied DMR which might be simple majority. However, things are different in a hierarchical set-up where decision making is sequential. If *b* has received a proposal on his desk there is neither a need for *c* to vote on the proposal already approved by *a* and *b*, nor –and this is more important- *c* will even know about the fact that *b* has a proposal on his desk. Again the same applies for *b* if *c* has a proposal on his desk. Formally speaking, what we are saying is that the DMRs contain two possible unanimity games going on in this hierarchy: one between *a* and *b* and one between *a* and *c*. To put into other words: we have to *truncate* the hierarchy two times, i.e. for each actor with a contact to the outer world. Whether $\mathcal{W} = \{ab\}$ or $\mathcal{W} = \{ac\}$ is played and whether *b* or *c* will be excluded from the decision-making process, depends on the fact whether *b* or *c* will receive the proposal presented to the hierarchy.

The same kind of reasoning applies for the DMRs $\tau = 1, 2, 3$. The only difference to the rules discussed above is, that also *a* has contact to the outer world and, therefore, can make decisions on his own, i.e. we have additionally $\{a\} \in \mathcal{W}$ and, thus, *a* is a dictator. However, nothing changes with regard to our argument concerning the non-existence of the grand coalition *N*. Instead of two we have now three possible unanimity games, with the additional game being $\mathcal{W} = \{a\}$.

The whole phenomenon just described can also be put into the context of abstention. Braham & Steffen (2002) suggest a framework for taking into account abstention for originally binary decisions by introducing the idea of 'possible games'. Depending on the 'abstentions of the actors $i \in N$ different games are played with sets of actors being subsets of *N*, i.e. '*N* minus the abstainers'. The same applies here: we have games played among subsets of actors. However, the difference lies in the interpretation of the term 'abstention'. While the literature on abstention implicitly presupposes abstention to be voluntary, in our case we encounter the fact that abstention is 'decreed'.

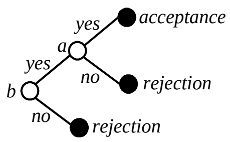
This amounts to the following insight: for each actor $i \in N$ with a contact to the outer world we have a truncated hierarchy obtained by deleting $N / \left\{ \hat{\mathcal{S}}^{-1}(i) \cup \{i\} \right\}$ and an associated possible game with $\hat{N} = \left\{ \hat{\mathcal{S}}^{-1}(i) \cup \{i\} \right\}$ being the set of actors. Even the existing measures could be modified to take this into account by applying the idea of an expected power measure based on an ensemble of possible games (see Braham and Steffen 2002), they suffer from a second pathology which a measure that can be represented by a simple game cannot handle.

Consider again Example 6 applying τ = 4, 5, 6. Let us focus on the possible game $\hat{\mathcal{W}}=\{ab\}$. As shown in Table 7 four different action profiles *seem* to be possible. But again, we have to emphasise that this ignores that decision-making is sequential. For τ = 4, 5, 6 only *b*, but not *a*, can receive a proposal from the outside world. If *b* receives a proposal and approves it, he will pass it on to *a* for his approval. Depending on *a*'s decision the collective decision will be either an acceptance or a rejection. However, if *b* decides to reject the proposal the collective decision is already made without *a* having to make a decision on the proposal. Thus, *a* is excluded from the decision-making by the decision of *b*. The consequence of this insight is that we do not have two distinct cases with *b* rejecting and *a* either approving or rejecting, but only one in which *b* rejects. This can be shown as well using a tree of a sequential game form as given by Figure 3.[19]

Table 8 Coalitions, Divisions and Swings in Fig. 1c with Truncation for τ = 4, 5, 6

$\hat{N}$	a_h			$o \in \mathbb{O}$	Swings		
	a	*b*	*c*		*a*	*b*	*c*
$\{ab\}$	*yes*	*yes*		*acceptance*	1	1	
	yes	*no*		*rejection*		1	
	no	*yes*		*rejection*	1		
	no	*no*		*rejection*			
$\{ac\}$	*yes*		*yes*	*acceptance*	1		1
	yes		*no*	*rejection*			1
	no		*yes*	*rejection*	1		
	no		*no*	*rejection*			
$\eta(\hat{f})$					4	2	2

Figure 3 Tree of a Sequential Game Form for $\hat{N}=\{ab\}$ and $\mathcal{S}\in\mathcal{S}_D^N$ given by $\mathcal{S}(a)=\{b\}$ and $\mathcal{S}(b)=\varnothing$.



[19] For a formal definition of a sequential game form see Section 8.

To sum up, we have showed that the existing approaches to measure decision-making power in hierarchies suffer two major conceptual pathologies resulting from ignoring that decision-making in a hierarchy is sequential. Formally speaking: $\mathcal{W}$ cannot properly represent $(\tau, \mathcal{S})$.[20] Both pathologies have in common that actors are included into the power calculation which in a certain are way excluded from the decision-making process. Both types of exclusions can be distinguished by origin of the exclusion. In the first case the exclusion was caused by the outside world which decides which admissible actor in hierarchy will receive the proposal, while in the second case the exclusion results from the decision of an actor within the hierarchy. Thus we may call the first case an 'external' and second an 'internal' exclusion.

8. Analysing Power in Hierarchical Decision-making Bodies: A New Approach

The existing (traditionally) simple games based measures have turned out to be incapable to capture the properties of a sequential decision-making process. However, if we base the representation of a measure of decision-making power on the apparatus of a game form, we can extend the idea underlying the classical approaches to measure power under a simultaneous decision-making structure to allow it also to be sequential by moving from a SGF (with unconditional actions) to an *extensive* (or *sequential*) *game form*.[21]

Informally, an EGF can be seen to be more general than a SGF (with unconditional actions) as it allows a decision-making process also to be sequential.[22] It captures not only (i) what actions each actor can take and (ii) what the outcome is as a function of the actions taken by the actors, but also (iii) who moves when and (iv) what actors know when they move. Formally, an extensive game form relies on the concept of a game tree being a specific interpretation of a digraph:[23]

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[20] Note, that even an approach making use of Nowak & Radzik's (1994) generalized characteristic function, which take into account the sequential nature of a decision-making process cannot solve this problem.

[21] We are aware of the fact that a SGF which allows for conditional actions is a legitimate alternative to an EGF, i.e. that it is not necessary to move from an SGF to an EGF to obtain our results.

[22] Note, that one might see this differently as one can also argue that an EGF is more specific than a SGF. For this purpose we have to start with the position that a SGF with unconditional actions also allows a decision-making process to be sequential, but lacks information about the sequence. Thus, a SGF permits more than one sequential structure, while the EGF specifies one particular structure, and, hence, contains more information which implies that it is more specific.

[23] For more details about extensive game forms we refer the reader to Koplin (1988, 1989) and for extensive form games to the original contributions by Kuhn (1953) and Selten (1975) or the standard textbooks such as Aliprantis & Chakrabarti (2000), Kreps (1990), Mas-Colell *et al*. (1995) or Myerson (1991).

Let $N = \{1,...,n\}$ be the set of actors of a collective decision making body and $N^* =_{def} N \cup n$ where n denotes 'Nature' which acts randomly, i.e. n's actions represent the exogenous effects of the outer world on *N*.

We call $\mathcal{T}$ a tree of a game form if (i) each non-terminal node is owned by exactly one $i \in N^*$ and (ii) to each terminal node $k \in \mathcal{V}$ of the tree an outcome o belonging to the *outcome set* $\mathbb{O}$ is assigned, i.e. $o(k) \in \mathbb{O}$. If $j \in \mathcal{V}$ is owned by an $i \in N$, then such a node is called a *decision-making node* and if it is own by n it is called a *chance node*. Moreover, we denote the set of nodes owned by an actor *i* by $\mathcal{V}_i$.

In a tree of a game form, a set of nodes $\hbar$ is called a n *information set* for an actor *i*, $\hbar_i$, if (i) all nodes of $\hbar_i$ are owned by *i*, (ii) no node of $\hbar_i$ is related to any other node of $\hbar_i$, i.e. if $h,k \in \hbar_i$, then *h* is neither an ancestor nor a successor of *k*, and (iii) all nodes of $\hbar_i$ are equivalent for *i*, i.e. the number and 'type' of edges starting from each node $h \in \hbar_i$ is the same and it does not matter for *i* if there is a rearrangement of the set of edges $\{e_1^h,...,e_m^h\}$ starting form each node $h \in \hbar_i$ such that $\forall h,k,l : e_l^h \approx e_l^k$. Moreover, let us denote the set of all information sets of an actor *i* by $\mathcal{H}_i$, i.e. $\mathcal{H}_i =_{def} \{\hbar_{i1},..,\hbar_{ip}\}$.

The intuition behind the notion of $\hbar_i$ is that an actor *i* in an decision-making process might know that he must make the 'next decision', but due to his lack of information of the 'history' of the decision-making process *i* does not know the node $h \in \hbar_i$ where he must make his decision; *i* only knows that the node $h \in \hbar_i$.

Note, that in our context we will make use of two extreme assumptions on $\mathcal{H}_i$: (i) $\forall i \in N : \#\mathcal{H}_i = 1$ and $\hbar_{i1} = \mathcal{V}_i$, i.e. there is one information set containing all nodes owned by *i* (imperfect information) and (ii) $\forall i \in N : \#\mathcal{H}_i = \#\mathcal{V}_i$ and $\forall j \in \mathcal{V}_i : j = \hbar_{ik}$, i.e. information sets are singletons and there is one set for each node owned by *i* (perfect information). Moreover, if the set of nodes of a tree $\mathcal{T}$ of a game form is *N**, i.e. it includes n, we follow the convention that nodes belonging to nature are always singleton information sets.

Now we can characterise an EGF as a game tree such that the decision-nodes have been partitioned into information sets that belong to the actors.

Moreover, by $\mathcal{C}_i$ we denote the choice function of an actor $i \in N^*$ in an EGF and by $\mathcal{C}$ the set of choice functions for all actors. For *i* owning the elements of $\mathcal{V}_i$ it is a function $\mathcal{C}_i : \mathcal{V}_i \rightarrow \mathfrak{Q}_i$ where $\mathfrak{Q}_i$ denotes the set of nodes of the tree where $\forall k \in \mathfrak{Q}_i$: *k* is a successor of *j*, i.e. $\mathcal{C}_i$ is a mapping from every node $j \in \mathcal{V}_i$ to the nodes that can be reached by *i* from each node *j*. We say that *i* '*has chosen*', if *i* has to make a move in a node $j \in \mathcal{V}_i$ which other node to reach and *j* has decided for a node $h \notin \mathcal{V}_i$ that he can reach being in *j*.

For all $i \in N$ we call such a choice an action of *i* in *j* denoted by a_{ij}. Furthermore, the set of all actions available to *i* at a node $j \in \mathcal{V}_i$ is called the action set of *i* at *j*, denoted by A_{ij}, i.e. $A_{ij} =_{def} \{a_{ij1},...,a_{ijq}\}$, and the set of all actions available to *i* at all nodes in $\mathcal{V}_i$ is denoted by A_i, i.e. $A_i =_{def} \{A_{ij} \mid j \in \mathcal{V}_i\}$.

If instead the nature n moves, their moves – if they are non-fictitious – are not actions as those imply a deliberative element (Pearl 2000, 108 ff.). They are just moves (determining which 'real' actor is allowed to choose next) that follow an probability distribution resulting from a function $p: \mathcal{H}_n \times A_n \to [0,1]$ assigning probabilities to the 'actions' at information sets where the nature moves, satisfying $\forall \hbar_{nk} \in \mathcal{H}_n : \sum \{a_{nj} \in A_{nj} \mid j \in \hbar_{nk} : p(a_{nj})\} = 1$.

Next, let us define an action profile in an EGF as $a_{\hbar} =_{def} \mathcal{P}(r,k)$ with *r* being the root and *k* being a terminal node. Then the set of all action profiles is $\mathcal{A}$ is the set of all possible paths from the root to one of the terminal nodes. Thus, an action profile is a vector of individual actions a_{ij} of a subset of actors and the moves of the nature which together form a path $\mathcal{P}(r,k)$. Moreover, let $\mathcal{A}_i \subset \mathcal{A}$ be set subset of all action profiles that contain an action of actor *i*.

Finally, notice that if the set of actors of an EGF is *N* and $\forall i \in N : \#\mathcal{H}_i = 1$ and $\hbar_{i1} = \mathcal{V}_i$, then an extensive game form is equivalent to a SGF as decision-making is de facto simultaneous.

At this point let us return to our Example 6 under τ = 4, 5, 6 which we can now be represented as an EGF with $N^* = \{a,b,c,n\}$, $\forall j \in \mathcal{V}_i : j = \hbar_{ik}$, i.e. perfect information, $\forall j \in \mathcal{V}_i, i \in N : A_{ij} = \{yes, no\}$, i.e. when we analyse the decision-making process in a hierarchy an actor may own more than one decision node, even he owns only one node in the dominance structure, and $\mathbb{O} = \{acceptance, rejection\}$. Given the tuple $(N^*, \tau, \boldsymbol{\mathcal{S}})$ we obtain $\mathcal{C}$ and A_i as given by Table 9 from which we can derive the action profiles and their related outcomes given by Table 10.

Table 9 Choice Functions and related Actions Sets for Example 6 under τ = 4, 5, 6

$j \in \mathcal{V}_i$	$\mathcal{Q}_i$	A_{ij}
$j \in \mathcal{V}_a$	{*acceptance*, *rejection*}	{*yes*, *no*}
$j \in \mathcal{V}_b$	{*a*, *rejection*}	{*yes*, *no*}
$j \in \mathcal{V}_c$	{*a*, *rejection*}	{*yes*, *no*}
$j \in \mathcal{V}_n$	{*b*, *c*}	-

Table 10 Actions Profiles and Related Outcomes for Example 6 under $\tau = 4, 5, 6$

$a_h \in \mathcal{A}$	$o \in \mathcal{O}$
(yes_a, yes_b, *b is allowed to choose*$_n$)	*acceptance*
(yes_a, yes_c, *c is allowed to choose*$_n$)	*acceptance*
(no_a, yes_b, *b is allowed to choose*$_n$)	*rejection*
(no_a, yes_c, *c is allowed to choose*$_n$)	*rejection*
(no_b, *b is allowed to choose*$_n$)	*rejection*
(no_c, *c is allowed to choose*$_n$)	*rejection*

What is left to be specified is p, i.e. the probability function determining the nature's moves. If nature determines which actor with a contact to the outer world will obtain a proposal on his desk, we have no (structural) information on the odds for the actors in question. Hence, we can apply the principle of insufficient reason of classical probability theory as a heuristic which assigns equal probability to all admissible 'atomic events', in the present case all, to all actors with a contact to the outer worlds who might receive a proposal on their desk:

$$prob\left(i \text{ receives a proposal} \middle| i \text{ has contact to the outer world}\right) = \frac{1}{g}$$

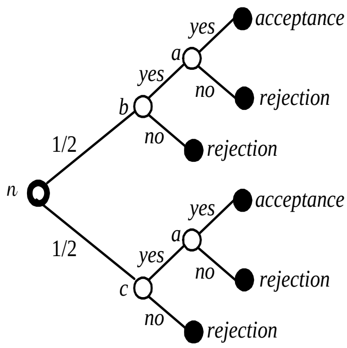
with g being the number of actors $i \in N$ with a contact to the outer world. In the absence about any information about the outside world this appears to be legitimate as we fulfil the condition that we have a finite probability space consisting of finitely many clearly distinguished indivisible 'atomic events' (Felsenthal & Machover 2003).

Now, we have everything together to give a proper graphical representation of the decision-making process which is given by Figure 4.

The next step is now to ascribe power to the actors in this EGF. Let us start with examination of *a*'s power. As we can see from Table 10 there are two instances, one together with *b* and one with *c*, for which *a* can either ensure a '*rejection*' or an '*accpetance*' of a proposal by, ceteris paribus, changing his own action from '*yes*' to '*no*' or vice versa, i.e. *a* has two negative and two positive swings. Thus, there is no difference with respect to the existing measures. However, things are different if we turn our view to *b* and *c*. As *b* and *c* are in symmetric position we can restrict ourselves to *b*. What *b* has in common with *a* is that if *b* has opted for '*yes*' and would switch to '*no*' he can, ceteris paribus, ensure a 'rejection' of the proposal, i.e. their negative swings are identical, if the ceteris paribus clause is applied. On the other hand, if *b* has originally chosen the '*no*'-

action and would switch to the 'yes'-action, he cannot, ceteris paribus, ensure an '*acceptance*', i.e. *b* has no swing in the usual sense, but *b* is still able to alter the outcome in some way, i.e. *b* can ensure that there is no definite rejection of the proposal. In order to distinguish this type of swing from the former on which both have in common, let us refer to the latter as a '*weak*' and to the former as a '*strong'* swing.

Figure 4 EGF for Example 6 under $\tau = 4, 5, 6$



For an illustration of the difference between the swings owned by *a* and *b* assume for the moment that the outcome set is no longer binary, but the interval $[0,1]$ where a higher value stands for a higher degree of acceptance of a proposal, i.e. '0' stands for a straight forward rejection and '1' for an acceptance without any reservation. Next assume that one actor has an action set which allows him by altering his own action to pick out any value within the interval as a collective decision, while another actor can only either pick out '0' or the whole interval. Then, we would say that the first actor is more powerful than the latter. Now, let us return to our binary set-up. Here this means that the first actor (which is our *a*), can determine whether the outcome will be '0' or '1', while the second actor (which are our *b*) can only decide among '0' and $\{0,1\}$. Thus, for the same reason as before, we would say that *a* can be regarded to be more powerful than *b* as *a*'s swings include more ability than *b*'s.

Going back to our original example we can now say that *a* owns two positive and two negative strong swings, while *b*, and by symmetry also *c*, owns one positive weak and one negative strong swing. From this it is obvious that *a* is more powerful than *b* and *c* and that *b* and *c* have equal power.

However, in order to allow for comparisons between different decision-making situations we require an approach that takes into account the presence of nature, if nature exists, i.e. that either *b* or *c* will right away excluded from the decision-making process (internal exclusion is already taken into account via the action profiles). We obtain this by weighting the swings on each branch of the tree of the game form with the likelihood that nature will choose this branch, i.e. $p(a_n \in a_h)$, and setting $p(a_n \in a_h) = 1$ if nature does not exist. For our example above this is equivalent with (i) identifying the branches of the tree of the game form which have an actor with a contact to the outer world as a root and (ii) weighting the swings on each of these branches with the inverse of the number of such actors, i.e. $\forall a_n \in A_n : p(a_n \in a_h) = 1/g$. However, as we will see later, this equivalence does not always hold. Thus, as a natural analogue to $\eta_i(f)$ we obtain:

$$\text{(4)} \qquad \begin{aligned} \eta_i(N^*, \tau, \mathcal{S}) =_{def} & \sum \{ a_h \in \mathcal{A} \mid i \text{ is a strong swinger in } a_h : p(a_n \in a_h) \} + \\ & \sum \{ a_h \in \mathcal{A} \mid i \text{ is a weak swinger in } a_h : (1-\varepsilon) \bullet p(a_n \in a_h) \} \end{aligned}$$

with $0 < \varepsilon < 1$ to take into account the nature of weak swings.[24]

From (4) we obtain the analogue to the Banzhaf measure $\beta'_i(f)$ by weighting each swing with the number of actions profiles of which *i* is a member:[25]

$$\text{(5)} \qquad \begin{aligned} \beta'_i(N^*, \tau, \mathcal{S}) =_{def} & \sum \{ a_h \in \mathcal{A} \mid i \text{ is a strong swinger in } a_h : p(a_n \in a_h) / \#\mathcal{A}_i \} + \\ & \sum \{ a_h \in \mathcal{A} \mid i \text{ is a weak swinger in } a_h : (1-\varepsilon) \bullet p(a_n \in a_h) / \#\mathcal{A}_i \} \end{aligned} .$$

Unfortunately, we will see in Section 9, that even this extension of the Banzhaf measure to sequential decision-making processes appears to be natural, the results for our examples put the denominator of the measure into question und suggest a rejection of this measure.

Let us wind up this section with an observation from the results of Example under τ = 4, 5, 6 comparing the existing approaches with our approach. Comparing the score and the Banzhaf measures (for the results of our approach see by Table 13) we find that the existing approaches tend (i) to overestimate the power differentials between actors, i.e. in this case between *a* and the other actors, and (ii) to ascribe too much power to the top. We stipulate that this holds in general, as can also observe this tendency for our remaining examples.

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[24] Note, that for the decision-making procedures τ considered in this paper it is not absolutely necessary to specify the value of ε as these procedures are (i) bottom-up procedures which (ii) always require the presence of the top for an approval. (i) ensures that the top is the only one who owns a positive strong swing, while (ii) guarantees that the top has never less swings than any other actor.

[25] Note, that in order to keep the exposition of our paper traceable we do not suggest an analogue to the Shapely-Shubik index here and leave this to future research.

9. Some Properties of Decision-Making Power in Hierarchies

In order to illustrate some basic properties of decision-making power in hierarchies if the sequential nature of the decision-making process is explicitly taken into account and to discuss the usefulness of the suggested analogue of the Banzhaf measure, let us apply the above score and measure to examples 4 to 7. The results for examples 4 to 6 are given by the tables 11 to 13.

Table 11 Decision-making Power in Fig. 1a (Example 4)

τ	1 – 3	4 – 6
$\eta(N^*,\tau,\mathcal{S})$	$(2.00, 1.00-0.50\varepsilon)$	$(2.00, 2.00-\varepsilon)$
$\beta'(N^*,\tau,\mathcal{S})$	$(0.50, 0.33-0.17\varepsilon)$	$(1.00, 0.67-0.33\varepsilon)$

Table 12 Decision-making Power in Fig. 1b (Example 5)

τ	1	2 – 3
$\eta(N^*,\tau,\mathcal{S})$	$(2.67, 0.67-0.33\varepsilon, 0.67-0.33\varepsilon)$	$(2.00, 1.33-0.67\varepsilon, 0.67-0.33\varepsilon)$
$\beta'(N^*,\tau,\mathcal{S})$	$(0.33, 0.10-0.05\varepsilon, 0.13-0.07\varepsilon)$	$(0.33, 0.22-0.11\varepsilon, 0.17-0.08\varepsilon)$

τ	4	5 – 6
$\eta(N^*,\tau,\mathcal{S})$	$(2.00, 0.00, 2.00-\varepsilon)$	$(2.00, 2.00-\varepsilon, 2.00-\varepsilon)$
$\beta'(N^*,\tau,\mathcal{S})$	$(1.00, 0.00, 0.60-0.20\varepsilon)$	$(1.00, 0.67-0.33\varepsilon, 0.50-0.25\varepsilon)$

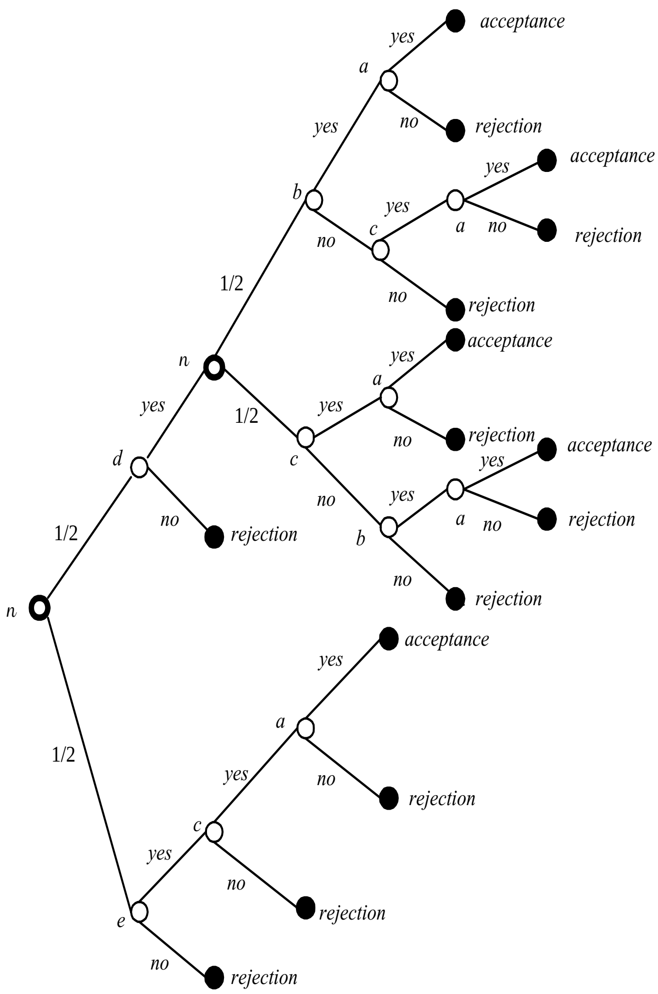
Table 13 Decision-making Power in Fig. 1c (Example 6)

τ	1 – 3	4 – 6
$\eta(N^*,\tau,\mathcal{S})$	$(2.00, 0.67-0.33\varepsilon, 0.67-0.33\varepsilon)$	$(2.00, 1.00-0.50\varepsilon, 1.00-0.50\varepsilon)$
$\beta'(N^*,\tau,\mathcal{S})$	$(0.33, 0.22-0.11\varepsilon, 0.22-0.11\varepsilon)$	$(0.50, 0.33-0.17\varepsilon, 0.33-0.17\varepsilon)$

Finally, let us investigate Example 7 which confronts us with another specific feature of sequential of decision-making procedures. This occurs if (i) we allow an actor to be directly dominated by more than one other actor and (ii) we are concerned with τ = 5. Under these conditions an actor has than more than one path to the top each being sufficient for a collective approval of a proposal, if all actors along the path choose the '*yes*'-action.

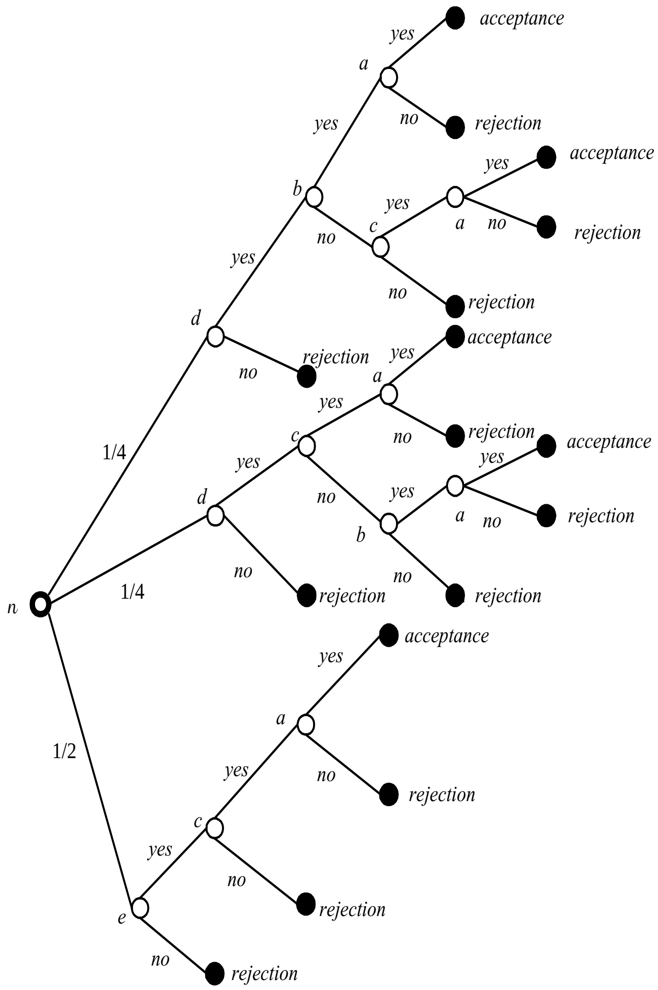
While the analysis for the branch starting with actor *e* is the same as in Example 5, the branch starting with *d* has a novel feature. Due to the fact that *d* has two paths to the top *a*, one via *b* and one via *c*, neither *b* or *c* are necessary for the approval of a proposal arriving at *d*'s desk. The actor which *d* will ask first is always not essential for the outcome, as *d* has always the option to ask the other actor who dominates him for approval. This also implies that *d* has not to care whether he should first ask *b* or *c* as there is always the option to go to the other actor, i.e. for the analysis of the game form we can say that *d* is indifferent between *b* and *c*. Therefore, we can stay with our initial structure of the game form that each actor has two actions, i.e. '*yes*' and '*no*', by introducing nature again. Here n decides whether *b* or *c* will be asked for their opinion, if *d* has opted for the '*yes*'-action. Figure 5 illustrates the corresponding EGF.

Figure 5 EGF for Example 7 under $\tau = 5$



Note, that the EGF given by Figure 5 can be simplified by integrating both nodes owned by the nature into a single node as shown by Figure 6. It is easy to proof via stepwise backward induction that the structure of both EGFs is (strategically) equivalent.[26]

Figure 6 Equivalent EGF for Example 7 under $\tau = 5$



Now, we can also calculate Example 7. The results are given by Table 14.

[26] The basic idea of the proof which we owe to Marlies Ahlert is to show that a situation in which an actor has two choices and one of them is followed by a choice of the nature is (strategically) equivalent to a situation in which the nature moves first. After one has shown that this holds, one has to apply this idea until one reaches the first node owned by nature. If it is reached, both nodes can be integrated into one.

Table 14 Decision-making Power in Fig. 2 (Example 7)

τ	1
$\eta(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(2.80, 0.40-0.20\varepsilon, 0.40-0.20\varepsilon, 0.60-0.20\varepsilon, 0.60-0.20\varepsilon)$
$\beta'(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(0.20, 0.08-0.04\varepsilon, 0.04-0.03\varepsilon, 0.24-0.08\varepsilon, 0.12-0.04\varepsilon)$
τ	2
$\eta(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(2.40, 0.60-0.30\varepsilon, 1.00-0.50\varepsilon, 0.60-0.20\varepsilon, 0.40-0.20\varepsilon)$
$\beta'(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(0.20, 0.09-0.04\varepsilon, 0.10-0.05\varepsilon, 0.10-0.03\varepsilon, 0.10-0.05\varepsilon)$
τ	3
$\eta(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(2.00, 0.80-0.40\varepsilon, 1.20-0.60\varepsilon, 0.40-0.20\varepsilon, 0.40-0.20\varepsilon)$
$\beta'(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(0.20, 0.11-0.06\varepsilon, 0.12-0.06\varepsilon, 0.08-0.03\varepsilon, 0.10-0.05\varepsilon)$
τ	4
$\eta(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(3.00, 0.00, 0.00, 1.50-0.5\varepsilon, 1.50-0.5\varepsilon)$
$\beta'(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(0.38, 0.00, 0.00, 0.60-0.20\varepsilon, 0.30-0.10\varepsilon)$
τ	5
$\eta(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(3.00, 1.00-0.75\varepsilon, 2.00-1.25\varepsilon, 1.50-0.50\varepsilon, 1.00-0.50\varepsilon)$
$\beta'(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(0.50, 0.25-0.19\varepsilon, 0.29-0.18\varepsilon, 0.25-0.08\varepsilon, 0.25-0.13\varepsilon)$
τ	6
$\eta(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(2.00, 1.00-0.50\varepsilon, 2.00-\varepsilon, 1.00-0.50\varepsilon, 1.00-0.50\varepsilon)$
$\beta'(N^*,\tau,\boldsymbol{\mathcal{S}})$	$(0.50, 0.25-0.13\varepsilon, 0.25-0.25\varepsilon, 0.20-0.10\varepsilon, 0.25-0.13\varepsilon)$

Based on the results given by tables 11 to 14 Table 15 provides a summary of the resulting power rankings. From Table 15 it is easy to see that the power ranking in a hierarchy heavily depends on τ such that power is neither necessarily monotone in rank nor in the number of subordinates.

Another important observation is that, in contrast, to the existing approaches, our power score and measure are not necessarily co-monotone. The reason for this is obvious. For the simultaneous case the number of action profiles of which an actor is a member is the same for all actors, which is no longer the case under a sequential set-up. However, as the effect of the weighting of the score in order to obtain a measure should be just to make the score cardinal comparable with results for other decision-making situations, the denominator (with the current interpretation) appears not only to be flawed under the sequential set-up, but has also, in principle, to be questioned.

Table 15 *Power Rankings*

Example	τ	Ranking for $\eta(N^*,\tau,\mathcal{S})$	Ranking for $\beta'(N^*,\tau,\mathcal{S})$
4	1-3	$a>b$	$a>b$
	4-6	$a>b$	$a>b$
5	1	$a>b=c$	$a>b=c$
	2-3	$a>b>c$	$a>b>c$
	4	$a>c>b$	$a>c>b$
	5-6	$a>b=c$	$a>b>c$
6	1-3	$a>b=c$	$a>b=c$
	4-6	$a>b=c$	$a>b=c$
7	1	$a>d=e>b=c$	$\varepsilon>0.5$: $a>d>e>b>c$
			$\varepsilon=0.5$: $a=d>e>b>c$
			$\varepsilon<0.5$: $d>a>e>b>c$
	2	$a>c>d>b>e$	$a>d>c=e>b$
	3	$a>c>b>d=e$	$a>c>b>e>d$
	4	$a>d=e>b=c$	$d>a>e>b=e$
	5	$a>c>d>e>b$	$\varepsilon<0.4$: $a>c>d>e>b$
			$\varepsilon=0.4$: $a>c=d>e>b$
			$0.4<\varepsilon<0.8$: $a>d>c>e>b$
			$\varepsilon=0.8$: $a>d>c=e>b$
			$\varepsilon>0.8$: $a>d>e>c>b$
	6	$a>c>b=d=e$	$a>b=e>d>c$

10. Concluding Remarks

Our analysis has shown that a simple game cannot properly represent decision-making processes if decision-making is sequential. Moreover, we argued that decision-making in hierarchies is rather sequential than simultaneous. Representing a decision-making process by a game form allows us to take into account a sequential nature of a decision-making process and to apply this set-up to analyse decision-making power in hierarchies which appears to non-monotone in rank and in the number of subordinates.

The way we addressed the issue is that we have extended the idea of a power ascription under a simultaneous set-up to a sequential one, i.e. we have ascribed power to actors in action profiles under a ceteris paribus clause and, then, we have aggregated over all action profiles of which an actor is a member. This is in line with our definition of power as a generic ability which is a conditional disposition that exists irrespectively whether it is exercised or not, i.e. it is not probabilistic. To put it in other words: our approach implies the assumption that we ascribe power to an actor $i \in N$ irrespectively of the likelihood that other actors $k \in N \setminus \{i\}$ will decide in a fashion that *i* has to exercise his swing(s). Taking into account such likelihood would mean to commit the exercise fallacy.

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