Measuring Economic Growth and Social Welfare with Lebesgue Integrals

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EPCS2006

Abstract. Clarke and Islam introduced an adjusted national income social welfare function to study the relationship between economic growth and social welfare. Their framework lacks however a notion by which this relationship is to be measured. In this paper, a simple extension is made by formulating a growth-welfare measure based on Lebesgue integral. Together with statistical estimation (or stochastic process) theory this measure implies a forecasting method. The developed methods can be used to measure and predict the need for the type of economic growth experienced with respect to social welfare.

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A brief outline of the content. Clarke and Islam (2004) investigated mappings of the type $g : T \longrightarrow \mathbb{R}$ and $f : T \longrightarrow \mathbb{R}$, where $g(\cdot)$ represents deflated national income, $f(\cdot)$ represents cost-adjusted national income social welfare, and T is a finite set representing time. To develop a sound measure, the mappings g and f should be formulated as a Lebesgue integrable functions. Thus, we consider a modified framework where the domains of the mappings are drawn from the class of measurable sets $\mathcal{T}$. Each time set $T \in \mathcal{T}$ can be taken to correspond, abusing the notation, the set T of individuals alive and active at that time set. Under this formulation, a set $T \in \mathcal{T}$ need not be finite.

It is not difficult to show that a mapping $D_T(g, f)$ defined as $g(x) - f(x)$, for $x \in T$, is Lebesgue integrable, if g and f are Lebesgue integrable. So given Lebesgue integrable mappings g and f, having suitable domains, the divergence mapping $D(g, f)$ can be integrated at distinct time sets $T, U \in \mathcal{T}$. If the two sets are of equal Lebesgue measure $m(T) = m(U)$, we can then compare meaningfully the resulting values. This construction gives information about whether the desirability of the type of economic growth experienced has increased, decreased, or remained unchanged, and if there is such a change, how large it is.

An application of simple variational technique together with the Helly compactness theorem shows that there exist a non-linear least-square estimate for g. From this, an estimate for f can be computed. These estimates give a basis for an estimate for $\int_T D(g, f)$. Thus, we can estimate the desirability of the type of economic growth experienced w.r.t. social welfare. By extrapolating the structure beyond the current time set $T \in \mathcal{T}$, we can then make forecasts about the desirability of the type of economic growth w.r.t. social welfare.

This construction lends itself naturally to topological and variational questions when the design of an optimal growth-welfare policy for a nation is considered. Although particular formulations of these problems are beyond the scope of this paper, we briefly comment the general structure of some of these problems. To take an example, suppose $\mathcal{F}$ is the class of social welfare functions defined on some interval T, all nondecreasing, and equally bounded. Consider a net-social welfare problem, formulated as a direct variational problem $J : \mathcal{F} \longrightarrow \mathbb{R}$. Under what conditions there exist $f^* \in \mathcal{F}$ such that $J(f) = maximum$?

Work cited. Clarke, M.–Islam, S.M.N.: *Economic Growth and Social Welfare: Operationalising Normative Social Choice Theory.* Amsterdam: Elsevier, 2004.