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Consistent Constitutions

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Abstract

Constitution is usually a pair of rules (s, S) that are used in a voting situation. The rule s is used to vote about the existing alternatives and the other rule S is used to vote about changing the rule s to some other rule s'. In this paper we consider the question that was presented in Barbera and Jackson 2004: What kind of constitutions are likely to emerge as prominent ones if the constitutions contain more than just two rules? In a constitution that contains any number of rules the n:th rule is used to decide about the $n-1$ rule. We define a notion of consistency that requires that that the constitution is formed of such rules that for every n the rule $n-1$ cannot be changed when the n:th rule is used. As a result we show that all consistent constitutions contain the same rule from $n = 2$ onwards. This provides an explanation to the casual observation that the constitutions have usually only two rules.

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1 Introduction

Barbera and Jackson (2004) study the problem of which rules are used to decide about rules that are used to decide about rules and so on. The solution to this problem involves rules that have some kind of fixed point property or choose themselves. In particular, BJ study rules that they call self-stable. Such a rule maintains itself against other rules when the decision rule used is just that rule. It turns out that self-stable rules may not exist or there may be a multitude of them. For this reason they focus on self-stable constitutions where there is a rule that is used to decide on things and then there is a higher level rule that is used to change the underlying rule.

BJ consider a society with a finite number of agents but because it is rather involved to analyse we briefly sketch their setup next assuming that there is a continuum of agents.

In a society there exists a state of affairs a which is called the status quo. Then there exists an alternative state of affairs b which may be chosen to be the new status quo. The decision between a and b is be made by a vote in some future date. There exists a continuum of voters in the society, and each voter's preference is represented by a number $p_i \in [0, 1]$. This tells the probability that she will vote for the alternative b when the election comes. There is a finite number types of voters numbered so that $p_1 < p_2 < ... < p_n$. The measure of type i voters is α_i and the measure of all agents is unity, $\sum_{i=1}^{n} \alpha_i = 1$. With this specification there are exactly

$$B = \sum_{i=1}^{n} \alpha_i p_i \tag{1}$$

agents that vote for b when it is staged against the status quo a.

Utilities for voter j from status quo a and from the alternative b are given by

$$\begin{aligned} U_j (a|j \text{ favors } a) = 1, \\ U_j (a|j \text{ favors } b) = 0 \end{aligned} \tag{2}$$

$$\begin{aligned} U_j (b|j \text{ favors } a) = 0, \\ U_j (b|j \text{ favors } b) = 1. \end{aligned} \tag{3}$$

Definition 1 *A (voting) rule is a number* $s \in [0, 1]$.

The voting rule tells how many votes the alternative b has to get in order to replace the status quo a. We now determine the agents' induced preferences over voting rules. If $s \in [0, B]$ then the challenger b will be chosen and the agents' expected utilities are $u_i (s) = p_i$. If $s \in (B, 1]$ then the status quo remains and the agents' expected utilities are $u_i (s) = 1 - p_i$.

An interesting situation is one where the agents vote about voting rules. This situation can be thought to arise when a particular voting rule is in a status quo position and another rule is in the role of the challenger. Then the first rule is used to decide whether the challenger replaces it or not. Rules that maintain themselves against challengers are in some sense stable, and motivate the following definition where μ denotes the measure of agents

Definition 2 *(Barbera and Jackson, 2004). A rule s is self-stable if for all other rules s'*

$$\mu\{i : u_i(s') > u_i(s)\} < s.$$

In many instances there is a higher level rule that is used to change the rule that governs the choice between a and b. BJ call the pair of rules a constitution.

Definition 3 *A consitution is a pair of voting rules (s, S), where s is used to decide between a and b and S is to be used when a change of s to some other rule s' is proposed.*

Definition 4 *A constitution (s, S) is self-stable if for all other rules s'*

$$\mu\{i : u_i(s') > u_i(s)\} < S.$$

BJ show that unlike self-stable rules self-stable constitutions always exist. This definition of a constitution is, however, lacking (as mentioned in the footnote 6 on page 6 of BJ) in the sense that one can continue this process to its extreme and ask what kind of rules are used to change the second rule in a constitution and so on.

2 The Model

To study the situation where there is a rule that is used to decide about the lower level rule we introduce an n-constitution.

Definition 5 *An n-constitution is $(s_0, s_1, ..., s_n)$ where s_0 is used to decide between a and b, and s_{i+1} is used when a change of s_i to some other rule s'_i is proposed, where $i \in \{0, 1, ..., n-1\}$.*

With the focus on ordered tuples of rules where the previous rule can only be changed using the next rule, self-stability and stable sets are more complicated tools than we need. We expect a good n-constitution $(s_0, s_1, ..., s_n)$ to be such that any rule s_{i+1} should uphold rule the lower level rule s_i. We say that the situation between s_{i+1} and s_i is stable.

Definition 6 *An n-constitution is consistent if rule s_{i+1} upholds rule s_i $i \in \{0, 1, ..., n-1\}$.*

Next we determine n-constitutions that are consistent, and for this purpose we have to determine the agents' preferences over the higher level rules. We do not consider the cases where all the agents have probabilities that are less (or greater) than one half leaving these as exercises to the reader. Suppose that we have a situation where all voters with index $i \in \{1, ..., k\}$, have probabilities $p_i \leq \frac{1}{2}$ and for all voters with index $j \in \{k+1, ..., n\}$ have probabilities $p_j > \frac{1}{2}$.

Denote the measure of voters of types $i \in \{1, ..., k\}$ by $\underline{s}$ and the measure of voters of types $j \in \{k+1, ..., n\}$ by $1 - \underline{s}$. In the sequel we assume that $\underline{s} < 1 - \underline{s}$; the other case is handled in a completely analogous manner, and is left to the reader.

Level 1:

Let us assume first that $s_0 \in [0, B]$. In the schema below there is an agent's utility from the level-one rule as well as 'yes' or 'no' depending whether the level-zero rule is being upheld by the level-one rule. When the level-one rule is smaller than $\underline{s}$ both the proponents of low and high level-zero rule can change it. Consequently we do not know what the agents' exact utilities are but it is not absolutely certain that the level-zero rule will remain at s_0. Thus, we do not regard this situation as an acceptable one, or one where the constitutional arrangement is stable (in an informal sense).

When the level-one rule is between $\underline{s}$ and $1 - \underline{s}$ the proponents of large level-zero rules cannot change the level-zero rule and the proponents of small level-zero rule do not want to change it as it is already small. Consequently, in this case and the next one level-zero rules are upheld by the level-1 rule.

$$u_i(s_1) = \begin{cases} ? & if\ s_1 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_1 \leq 1 - \underline{s} & yes \\ p_i & if\ 1 - \underline{s} < s_1 & yes \end{cases}$$

Let us next assume that $s_0 \in (B, 1]$. The first case is just as before for the same reason. When the level-one rule is between $\underline{s}$ and $1 - \underline{s}$ the proponents of small rules can change the level-zero rule, which they will do, and the situation is not stable. Finally, when the level-one rule is larger than $1 - \underline{s}$ neither the proponents of small or large rules can change the level-zero rule, and this is a stable situation.

$$u_i(s_1) = \begin{cases} ? & if\ s_1 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_1 \leq 1 - \underline{s} & no \\ 1 - p_i & if\ 1 - \underline{s} < s_1 & yes \end{cases}$$

Level 2:

Now there are three different cases to consider, namely all those "paths" which are stable (i.e. with yes).

2.1
$s_0 \in [0, B]$
$s_1 \in (\underline{s}, 1 - \underline{s}]$

$$u_i(s_2) = \begin{cases} ? & if\ s_2 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_2 \leq 1 - \underline{s} & yes \\ p_i & if\ 1 - \underline{s} < s_2 & yes \end{cases}$$

Here the level-zero rule is small, and the level-1 rule is such that those who favour large rules cannot effect a change. If the level-2 rule is small, i.e. $s_2 \leq \underline{s}$, both those who favour small rules and in particular those who favour large rules can change the level-1 rule to their favour, and the situation is not necessarily stable. However, if the level-2 rule satisfies $\underline{s} \leq s_2$ those who favour large level-zero rules cannot change the level-one rule and also not the level-zero rule. Thus, there are two stable cases here.

2.2
$s_0 \in [0, B]$
$s_1 \in (1 - \underline{s}, 1]$

$$u_i(s_2) = \begin{cases} ? & if\ s_2 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_2 \leq 1 - \underline{s} & yes \\ p_i & if\ 1 - \underline{s} < s_2 & yes \end{cases}$$

The logic here is exactly as in case 2.1

2.3
$s_0 \in (B, 1]$
$s_1 \in (1 - \underline{s}, 1]$

$$u_i(s_2) = \begin{cases} ? & if\ s_2 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_2 \leq 1 - \underline{s} & no \\ 1 - p_i & if\ 1 - \underline{s} < s_2 & yes \end{cases}$$

The level-zero rule is large, and the level-1 rule is such that neither those who favour large rules nor those who favour small rules can change the level-zero rule. If the level-2 rule is small, i.e. $s_2 \leq \underline{s}$ then anything can happen and the situation is not necessarily stable. If the level-2 rule is such that $\underline{s} \leq s_2 \leq 1 - \underline{s}$ then those who favour small rules can change the level-1 rule which they proceed to do as the level-zero rule is large. Thus, the situation is not stable. Finally, if $1 - \underline{s} < s_2$ no-one can effect a change in the level-1 rule which is such that no-one can effect a change in the level-zero rule. Thus, the situation is trivially stable.

One can do the similar analysis to the level-3 rules and so on but it should be clear by now that nothing interesting happens on these levels but the same rules that turned out stable on level-2 remain stable on the subsequent levels. We can then state

Proposition 1 *The consistent n-constitutions, where $n = \infty$ is possible, are of the following form:* $(s_0, s_1, ..., s_n) \in [0, B] \times [\underline{s}, 1]^n$ *or* $(s_0, s_1, ..., s_n) \in (B, 1] \times [1 - \underline{s}, 1]^n$.

We remind that we have considered only the situation where the number of those who favour small level-zero rules is larger than those who favour large level zero rules, i.e. $\underline{s} < 1 - \underline{s}$. One type of consistent n-constitutions are such that the level-zero rule is small, and the rules of succeeding levels require more support than there are supporters of large level-zero rules. There are also consistent n-constitutions that uphold themselves because the rules are so large that the support of neither those who favour small level-zero rules nor the support of those who favour large level-zero rules is sufficient to change the rules. This way it is possible to have a consistent n-constitution where the level-zero rule is large even though there is less support for large than small level-zero rules. Notice also that all rules following the second level constitution rules are similar in the sence that they belong to the same set. This, in a way, provides an explanation to the fact that we only observe constitutions that have two layers of rules; one for making the decision and the other for changing the rule to make decisions.

BJ study a society with a finite number of agents, and it involved to analyse. However, it seems like a good conjecture that the consistent n-constitutions would be roughly similar to the case analysed here; from the level-1 on the rules should be big to ensure stability between the successive levels.

References

[1] Barbera, S. and Jackson, M. O.(2004): "Choosing How to Choose: Self-Stable Majority Rules and Constitutions", *Quarterly Journal of Economics* 119, 3, 1011 - 1048.

[2] Kultti, K and Miettinen, P. (2004): "Stable Set and Voting Rules" a manuscript.

The derivation here substantiates the claims before Proposition 1.

Level 3.

3.1.)

$s_0 \in [0, B]$
$s_1 \in (\underline{s}, 1 - \underline{s}]$

3.1.1.)

$s_2 \in (\underline{s}, 1 - \underline{s}]$

$$u_i(s_3) = \begin{cases} ? & if\ s_3 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_3 \leq 1 - \underline{s} & yes \\ p_i & if\ 1 - \underline{s} < s_3 & yes \end{cases}$$

3.1.2.)

$s_2 \in (1 - \underline{s}, 1]$

$$u_i(s_3) = \begin{cases} ? & if\ s_3 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_3 \leq 1 - \underline{s} & yes \\ p_i & if\ 1 - \underline{s} < s_3 & yes \end{cases}$$

3.2.)

$s_0 \in [0, B]$
$s_1 \in (1 - \underline{s}, 1]$

3.2.1.)

$s_2 \in (\underline{s}, 1 - \underline{s}]$

$$u_i(s_3) = \begin{cases} ? & if\ s_3 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_3 \leq 1 - \underline{s} & yes \\ p_i & if\ 1 - \underline{s} < s_3 & yes \end{cases}$$

3.2.2.)

$s_2 \in (1 - \underline{s}, 1]$

$$u_i(s_3) = \begin{cases} ? & if\ s_3 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_3 \leq 1 - \underline{s} & yes \\ p_i & if\ 1 - \underline{s} < s_3 & yes \end{cases}$$

3.3.)

$s_0 \in (B, 1]$

$s_1 \in (1 - \underline{s}, 1]$

$s_2 \in (1 - \underline{s}, 1]$

$$u_i(s_3) = \begin{cases} ? & if\ s_3 \leq \underline{s} & no \\ p_i & if\ \underline{s} \leq s_3 \leq 1 - \underline{s} & no \\ 1 - p_i & if\ 1 - \underline{s} < s_3 & yes \end{cases}$$

Consistent constitutions are one of the following form:

when $s_0 \in [0, B]$ then

i.)$[0, B] \times [\underline{s}, 1]^{\infty}$

when $s_0 \in (B, 1]$, then

ii.) $(B, 1] \times [1 - \underline{s}, 1]^{\infty}$.