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Strategic deficit and distorting taxation in a democracy

Very, very preliminary version, please do not quote

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Abstract:

In this paper the strategic use of budget deficits is analyzed under the assumptions that taxes are distorting and that governments are elected via majority voting. Persson's and Svensson's (1989) model for the explanation of different budget surpluses is extended by introducing public consumption in the first period. Furthermore the political constitution of each period is described as a result of majority voting which is modelled by a median voter model. The co-existence of a public and private sector enables us to explain the budget deficits as a result of political polarization with a left-wing party and a right-wing party having different preferences for the size of the public sector.

JEL: H61, H62

1. Introduction

Public debt has a long history in many countries. Since long time many economic explanations of public debt have been offered. Recently, political-economic explanations have gained more and more interest. In those models politicians, voters, voting procedures and institutional settings of democratic countries are analyzed with respect to their inherent tendency to run debts. Surveys are provided by Alesina and Perotti (1995), Persson and Tabellini (1999a, 1999b and 2000) and von Weizsäcker (1992).

An very interesting explanation is that a government chooses public debt strategically in order to alter the second period's composition or size of the public sector if the preferences of the incumbent are different from those of its successor. Decisive for this result are the assumptions that outstanding debt needs to be served at the end of the second government's term of office, e. g. that Ponzi games are ruled out effectively.

This strand of research has been mainly represented by the works of Tabellini and Alesina (1990) and Persson and Svensson (1989). In their two-period model Tabellini and Alesina explore a first-period median voter's incentive to run a budget deficit when there is uncertainty about the second-period median voter's preferences or when it is certain that the second-period median voter will favour another *composition* of government consumption. Tabellini and Alesina do not model the private sector at all. In addition to issuing or repaying public debt each government is assumed to have at its disposal a lump sum amount of money (tax revenues) in order to finance its expenditures. Such a setting does not allow determining the size of the public and the private sector. In Siebel (2005a and 2005b) I have modified this paper by endogenizing a private sector which is taxed in order to finance the public sector. However, these taxes are non-distorting.

Persson and Svensson (1989) offer a similar approach in which two different parties disagree over the *size* of public spending. If one party knows it will not be re-elected it may tempt to alter its successor's budget constraint by leaving a surplus which is different from the case when it would stay in office. Furthermore the authors introduce a private sector whose intertemporal decision on consumption and leisure is distorted by taxation on income. Since Persson and Svensson do not explicitly model a voting procedure it remains doubtful if their model allows the explanation of strategic budget policy in a democratic environment. Additionally, the formal setting of this model prevents a solution with budget deficits.

The following analysis aims at explaining budget deficits in a democracy by means of a two-period median voter model with distorting taxation. In contrast to the work of Siebel (2005a) the taxation of the private sector will be distorting, whereas the paper differs from Persson and Svensson (1989) by using a median voter approach and by allowing for public consumption in the first period.

The paper is organized as follows: first, the general properties of the models will be introduced in chapter 2. Chapter 3 examines the model under several assumptions about the median voters' preferences. Chapter 4 concludes.

2. The base model

2.1 Basic assumptions

The modelling of the private sector is mainly based on Persson and Svensson (1989). x_i is the consumption of a private good and l_i is labour in period $i=1, 2$. Any consumer has the intertemporal utility function $C^1(x_1) + H^1(l_1) + x_2 + H^2(l_2)$, with $C_{x_1}^1 > 0$, $C_{x_1, x_1}^1 < 0$ being the first derivatives of the utility function of first period's private consumption. $H^i(l_i)$ describes the 'utility' of labour in period $i=1, 2$ and the properties of that function are $H^i(1)=0$, $H_{l_i}^i(l_i) < 0$ and $H_{l_i l_i}^i(l_i) < 0$. Furthermore it is assumed that

$$(1) \quad H^1(l_1) = H^2(l_2) \quad \forall l_1 = l_2.$$

Each consumer earns an income of 1 per unit of labour. Hence his pre-tax income in period $i=1, 2$ is l_i and he earns an income of $\omega_i l_i$ after taxes with $\omega_i \in [0, 1]$. In absence of any interest rate a consumer can transfer money from one period to another and his intertemporal budget constraint reads

$$(2) \quad x_1 + x_2 = \omega_1 l_1 + \omega_2 l_2.$$

In each period the government provides the amount g_i of the single public good for free charge. Alternatively g_i can be seen as size of the public sector. The public good is funded by taxes revenues $(1 - \omega_i) \cdot l_i$, corrected by the budget balance (surplus or deficit) generated by a borrowing or lending on a foreign capital market, b in the period. Hence the public budget constraint of the first period's government ("government 1") is¹

¹ The model of Persson / Svensson (1989) can be seen as a special case, assuming that $\gamma_1 = -b$.

$$(3) \quad (1 - \omega_1) \cdot l_1 = g_1 - b =: \gamma_1.$$

The second period's government ("government 2") cannot borrow or lend on a capital market but has to serve its predecessor debt². Thus government 2's budget constraint reads

$$(4) \quad (1 - \omega_2) \cdot l_2 = g_2 + b =: \gamma_2.$$

At the beginning of period i , $i=1,2$ each consumer has the opportunity to vote on the amount of the public good. Whereas all consumers have the same preference for private consumption and leisure they differ with regard to their preference for the public good.

Consumer j , characterized by the preference parameter $\alpha^j \geq 0$ has the intertemporal utility

$$W^j(x_1, l_1, x_2, l_2, g_1, g_2, \alpha^j) = C^1(x_1) + H^1(l_1) - x_1 + \omega_1 l_1 + \omega_2 l_2 + H^2(l_2) + \alpha^j U(g_1) + \alpha^j U(g_2),$$

with $U(g_1) = U(g_2) \quad \forall g_1 = g_2$.

2.2 The second period

At the beginning of the second period consumer j maximizes his utility from private consumption and leisure,

$$(5) \quad \max_{l_2} V^2(l_2) = -x_1 + \omega_1 l_1 + \omega_2 l_2 + H^2(l_2) + \alpha^j U(g_2),$$

yielding the first order condition

$$(6) \quad V_{l_2}^2(l_2) = \omega_2 + H_{l_2}^2(l_2) = 0,$$

which can be totally differentiated to

$$(7) \quad \frac{dl_2}{d\omega_2} = L_{\omega_2}^2 = -\frac{1}{H_{l_2 l_2}^2} > 0.$$

l_2 is strictly increasing in the public variable ω_2 ³. Using (4) together with the implicit function theorem we get⁴

$$(8) \quad \frac{d\omega_2}{d\gamma_2} = \Omega_{\gamma_2}^2 = \frac{1}{(1 - \omega_2) \cdot L_{\omega_2}^2(\omega_2) - L^2(\omega_2)} < 0.$$

At least a few consumers vote on the policy variable γ_2 at the beginning of the second period, knowing their reaction function and the budget deficit b inherited from the first period.

Hence voter j calculates

² Empirical research by Bohn and Inman (1996) shows that stringent balanced budget rules have a significant influence on public spending in certain U. S. states. See also Poterba (1996). Another kind of budget restriction is the Maastricht treaty, which gives a strict ceiling for public debt within EU countries.

³ Because of linear utility in private consumption, private labour supply has no income effects and depends on wages only. See also Diamond (1998) and Atkinson (1990).

⁴ It is assumed, that the government always chooses the lowest tax rate that is required to fund public net expenditures γ_2 . Thus the chosen tax rate is always on the increasing side of the Laffer-curve.

$$\max_{\gamma_2} W^2(\gamma_2, \alpha_2^j) = -x_1 + \omega_1 l_1 + \Omega^2(\gamma_2) \cdot L^2[\Omega^2(\gamma_2)] + H^2\{L^2[\Omega^2(\gamma_2)]\} + \alpha_2^j U(\gamma_2 - b)$$

and his first order condition for a maximum is

$$(9) \quad W_{\gamma_2}^2 = \Omega_{\gamma_2}^2 \cdot l_2 + \alpha_2^j U_{g_2} = 0.$$

(9) is totally differentiated with respect to γ_2 , α_2^j and b and regarding (4) we obtain

$$(10a) \quad \frac{d\gamma_2}{d\alpha_2^j} = \frac{dg_2}{d\alpha_2^j} = -\frac{U_{g_2}}{\Omega_{\gamma_2\gamma_2}^2 \cdot l_2 + (\Omega_{\gamma_2}^2)^2 \cdot L_{\omega_2}^2 + \alpha_2^j \cdot U_{g_2g_2}} > 0$$

and

$$(10b) \quad \frac{dg_2}{db} = \frac{d\gamma_2}{db} - 1 = -\frac{\Omega_{\gamma_2\gamma_2}^2 \cdot l_2 + (\Omega_{\gamma_2}^2)^2 \cdot L_{\omega_2}^2}{\Omega_{\gamma_2\gamma_2}^2 \cdot l_2 + (\Omega_{\gamma_2}^2)^2 \cdot L_{\omega_2}^2 + \alpha_2^j U_{g_2g_2}} \in]-1, 0[.$$

(10a) shows that each voter j has a preferred value g_2^j for the public sector depending on the value of his preference parameter α_2^j . A sufficient condition for the sign of (10a) and (10b) is⁵

$$(11) \quad \Omega_{\gamma_2\gamma_2}^2 \cdot l_2 + (\Omega_{\gamma_2}^2)^2 \cdot L_{\omega_2}^2 < 0.$$

Furthermore we have $W_{\gamma_2}^2 = \Omega_{\gamma_2\gamma_2}^2 \cdot l_2 + (\Omega_{\gamma_2}^2)^2 \cdot L_{\omega_2}^2 + \alpha_2^j U_{g_2g_2} < 0$, which ensures that W^2 is single-peaked and thus the median voter's preferences are decisive for the election outcome (Black, 1948). Thus the size g_2^m of the public sector is given by a function $g_2^m = G^2(\alpha_2^m, b)$.

Following the interpretation of Persson and Svensson (1989) and Siebel (2005a) a median voter with high (low) preference for the public sector is associated with a left-wing (right-wing) party.

2.3 The first period

Next, we deal with consumer behaviour in the first period.

$$\max_{x_1, l_1} V^1(x_1, l_1, x_2, l_2, g_1, g_2, \alpha_1^j) = C^1(x_1) + H^1(l_1) - x_1 + \omega_1 l_1 + \omega_2 l_2 + H^2(l_2) + \alpha_1^j U(g_1) + \alpha_1^j U(g_2)$$

is consumer j 's calculus at the beginning of the first period. The first order conditions

$$V_{x_1}^1(x_1, l_1) = C_{x_1}^1(x_1) - 1 = 0 \text{ and}$$

$$(12) \quad V_{l_1}^1(x_1, l_1) = H_{l_1}^1(l_1) + \omega_1 = 0$$

imply $x_1 := \bar{x}_1 = \text{const.}$ as well as

⁵ $\Omega_{\gamma_2\gamma_2}^2 \cdot l_2 + (\Omega_{\gamma_2}^2)^2 \cdot L_{\omega_2}^2 < 0$ means that the marginal cost of public consumption is increasing.

$$(13) \quad \frac{dl_1}{d\omega_1} = -\frac{1}{H_{l_1}^1} > 0.$$

Invoking (3) and the implicit function theorem leads to⁶

$$(14) \quad \frac{d\omega_1}{d\gamma_1} = \Omega_{\gamma_1}^1 = \frac{1}{(1-\omega_1) \cdot L_{\omega_1}^1(\omega_1) - L^1(\omega_1)} < 0.$$

As l_1 is strictly monotone in ω_1 , which in turn is strictly monotone in γ_1 we are able to derive an indirect utility function $V^1(\gamma_1) = C^1(\bar{x}_1) + H^1[L^1(\Omega^1(\gamma_1))] - \bar{x}_1 + \Omega^1(\gamma_1) \cdot L^1(\Omega^1(\gamma_1))$.

Adding $\alpha_1^j U(g_1)$ and taking care of (3) yields voter j 's calculus on the size of the public sector in the first period⁷:

$$(15) \quad \max_{g_1} W^1(g_1, \alpha_1^j) = C^1(\bar{x}_1) + H^1\{L^1[\Omega^1(g_1 - b)]\} - \bar{x}_1 + \Omega^1(g_1 - b) \cdot L^1[\Omega^1(g_1 - b)] \\ + \Omega^2(\gamma_2) \cdot L^2(\Omega^2(\gamma_2)) + H^2(l_2) + \alpha_1^j U(g_1) + \alpha_1^j U(g_2).$$

(12) simplifies the first order condition

$$W_{g_1}^1(g_1, \alpha_1^j) = H_{l_1}^1 \cdot L_{\omega_1}^1 \cdot \Omega_{\gamma_1}^1 + \Omega_{\gamma_1}^1 \cdot l_1 + \omega_1 \cdot L_{\omega_1}^1 \cdot \Omega_{\gamma_1}^1 + \alpha_1^j \cdot U_{g_1} = 0$$

$$(16) \quad W_{g_1}^1(g_1, \alpha_1^j) = \Omega_{\gamma_1}^1 \cdot l_1 + \alpha_1^j \cdot U_{g_1} = 0,$$

and the second derivative is $W_{g_1 g_1}^1 = \Omega_{\gamma_1 \gamma_1}^1 \cdot l_1 + L_{\omega_1}^1 \cdot (\Omega_{\gamma_1}^1)^2 + \alpha_1^j \cdot U_{g_1 g_1}$. The sign of $\Omega_{\gamma_1 \gamma_1}^1 \cdot l_1 + L_{\omega_1}^1 \cdot (\Omega_{\gamma_1}^1)^2$ is ambiguous, but regarding (11) it can be assumed that

$$(17) \quad W_{g_1 g_1}^1 = \Omega_{\gamma_1 \gamma_1}^1 \cdot l_1 + L_{\omega_1}^1 \cdot (\Omega_{\gamma_1}^1)^2 + \alpha_1^j \cdot U_{g_1 g_1} < 0.$$

(17) is a sufficient condition for the single-peakedness of $W^1(g_1, \alpha_1^j)$ in (15). Thus the median voter's preferences for the size of the public sector are decisive and α_1^j can be replaced by α_1^m .

Total differentiation of (16) with respect to g_1 , b and α_1^m yields

$$(18a) \quad \frac{d\gamma_1}{db} = \frac{dg_1}{db} - 1 = -\frac{\alpha_1^m \cdot U_{g_1 g_1}}{\Omega_{\gamma_1 \gamma_1}^1 \cdot l_1 + L_{\omega_1}^1 \cdot (\Omega_{\gamma_1}^1)^2 + \alpha_1^m \cdot U_{g_1 g_1}} \in]-1, 0[$$

and

$$(18b) \quad \frac{dg_1}{d\alpha_1^m} = \frac{-U_{g_1}}{\Omega_{\gamma_1 \gamma_1}^1 \cdot l_1 + L_{\omega_1}^1 \cdot (\Omega_{\gamma_1}^1)^2 + \alpha_1^m \cdot U_{g_1 g_1}} > 0$$

⁶ For the sign of (14) see footnote 2.

⁷ With consumers' / voters' preferences being time consistent, the preference parameter α may also be written without period index. However, this index helps us to regard which period is affected.

due to (17). The size g_1^m of the public sector is a function $g_1^m = G^1(\alpha_1^m, b)$. With g_i and γ_i

being strictly monotone in b , (15) can be reduced to the single dimension b :

$$\begin{aligned} \max_b \tilde{W}^1(b, \alpha_1^m, \alpha_2^m) = & C^1(\bar{x}_1) + H^1 \left\{ L^1 \left[\Omega^1 \left(\Gamma^1(b, \alpha_1^m) \right) \right] \right\} - \bar{x}_1 + \Omega^1 \left(\Gamma^1(b, \alpha_1^m) \right) \cdot L^1 \left[\Omega^1 \left(\Gamma^1(b, \alpha_1^m) \right) \right] \\ & + \Omega^2 \left(\Gamma^2(b, \alpha_2^m) \right) \cdot L^2 \left[\Omega^1 \left(\Gamma^1(b, \alpha_2^m) \right) \right] + H^2 \left\{ L^2 \left[\Omega^2 \left(\Gamma^2(b, \alpha_2^m) \right) \right] \right\} \\ & + \alpha_1^m U \left(G^1(b, \alpha_1^m) \right) + \alpha_1^m U \left(G^2(b, \alpha_2^m) \right). \end{aligned}$$

After some transformations the first order condition reads

$$(19) \quad \tilde{W}_b^1(b, \alpha_1^m, \alpha_2^m) = \alpha_1^m \cdot U_{g_1} - \alpha_2^m \cdot U_{g_2} + (\alpha_1^m - \alpha_2^m) \cdot U_{g_2} \cdot G_b^2 = 0.$$

We assume, that $\tilde{W}_{bb}^1(b, \alpha_1^m, \alpha_2^m) < 0$, so that the function is single peaked in b and the value b^m that solves (19) leads to a maximum⁸.

3. Solutions for various combinations of α_1^m and α_2^m

In our further analysis we refer to the simple case $\alpha_1^m = \alpha_2^m$. In this case, the median voter will be same across both periods, e.g. the party being in office is sure to be re-elected at the beginning of the second period. Then (19) reduces to

$$(20) \quad \tilde{W}_b^1(b, \alpha_1^m, \alpha_2^m) = \alpha_1^m \cdot U_{g_1} - \alpha_1^m \cdot U_{g_2} = 0.$$

(20) implies $g_1^m = g_2^m$ and $\Omega_{\gamma_1}^1 \cdot l_1 = \Omega_{\gamma_2}^2 \cdot l_2$, which in turn yields $\gamma_1^m = \gamma_2^m$, $\omega_1^m = \omega_2^m$, $l_1 = l_2$ and $\bar{x}_1 = x_2$, as (1), (3) and (4) hold. Then $b^m = 0$.

If government 1 is certain to stay in office during the second period a balanced budget occurs. All public and private variables will be smoothed out in order to avoid intertemporal distortions.

Next, we examine whether $b^m < 0$, $b^m = 0$ or $b^m > 0$ if $\alpha_1^m \neq \alpha_2^m$. For doing so, we need the cross derivative of $\tilde{W}^1(b, \alpha_1^m, \alpha_2^m)$ with respect to b and α_2^m . It is assumed that

⁸ The second derivative is $\tilde{W}_{bb}^1(b, \alpha_1^m, \alpha_2^m) = \alpha_1^m U_{g_1 g_1} G_b^1 - \alpha_2^m U_{g_2 g_2} G_b^2 + (\alpha_1^m - \alpha_2^m) \left[U_{g_2 g_2} (G_b^2)^2 + U_{g_2} G_{bb}^2 \right]$ and can be transformed into

$\tilde{W}_{bb}^1(b, \alpha_1^m, \alpha_2^m) = \alpha_1^m \cdot \left[U_{g_1 g_1} G_b^1 + U_{g_2 g_2} (G_b^2)^2 + U_{g_2} G_{bb}^2 \right] - \alpha_2^m \cdot \left[U_{g_2 g_2} G_b^2 \cdot (1 + G_b^2) + U_{g_2} G_{bb}^2 \right]$. The signs of all summands except $U_{g_2} G_{bb}^2$ are clear and we have $U_{g_1 g_1} G_b^1 + U_{g_2 g_2} (G_b^2)^2 < 0$ and $U_{g_2 g_2} G_b^2 \cdot (1 + G_b^2) > 0$.

$\tilde{W}^1(b, \alpha_1^m, \alpha_2^m)$ is single-peaked in b and we know that $\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) = 0$ if $\alpha_1^m = \alpha_2^m$. If $\tilde{W}_{b\alpha_2^m}^1(b, \alpha_1^m, \alpha_2^m) \neq 0$, then $\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) \neq 0$ in case of $\alpha_1^m \neq \alpha_2^m$. The strict concavity of $\tilde{W}^1(b, \alpha_1^m, \alpha_2^m)$ in b ensures that $b^m \neq 0$ and thus $b^m \begin{cases} < \\ > \end{cases} 0$ if and only if $\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) \begin{cases} < \\ > \end{cases} 0$. More generally, it can be said that

$$(21) \quad \text{sign } b^m = \text{sign } \tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m).$$

The cross derivative of \tilde{W}^1 with respect to b and α_2^m is

$$\tilde{W}_{b\alpha_2^m}^1 = -U_{g_2} - \alpha_2^m \cdot U_{g_2g_2} \cdot G_{\alpha_2^m}^2 - U_{g_2} \cdot G_b^2 + (\alpha_1^m - \alpha_2^m) \cdot [U_{g_2g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2 + U_{g_2} \cdot G_{b\alpha_2^m}^2].$$

From (10a) and (10b) we see that $-U_{g_2} - \alpha_2^m \cdot U_{g_2g_2} \cdot G_{\alpha_2^m}^2 - U_{g_2} \cdot G_b^2 = 0$ and thus $\tilde{W}_{b\alpha_2^m}^1$ can be written as follows:

$$(22) \quad \tilde{W}_{b\alpha_2^m}^1 = (\alpha_1^m - \alpha_2^m) \cdot [U_{g_2g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2 + U_{g_2} \cdot G_{b\alpha_2^m}^2].$$

The complex induction of $G_{b\alpha_2^m}^2$ is shown in the appendix. The sign of $(\alpha_1^m - \alpha_2^m)$ can be easily determined, but the sign of $[U_{g_2g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2 + U_{g_2} \cdot G_{b\alpha_2^m}^2]$ is ambiguous as the sign of the cross derivative $G_{b\alpha_2^m}^2$ is unclear⁹. However, the signs of all other components of [...] are clear, leading to the conclusion

$$(23) \quad [U_{g_2g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2 + U_{g_2} \cdot G_{b\alpha_2^m}^2] \begin{cases} < \\ = \\ > \end{cases} 0, \text{ if and only if } G_{b\alpha_2^m}^2 \begin{cases} < \\ = \\ > \end{cases} - \frac{U_{g_2g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}} < 0.$$

$U_{g_2} > 0$ and $U_{g_2g_2} < 0$ together with (10a) and (10b) imply that $-\frac{U_{g_2g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}} < 0$.

Table 1 – 3 sum up the various cases which lead to different signs of $\tilde{W}_{b\alpha_2^m}^1$.

⁹ This is owed to the fact that the third and fourth derivatives of $H^2(l_2)$ are included in $G_{b\alpha_2^m}^2$

	$\alpha_1^m > \alpha_2^m$	$\alpha_1^m = \alpha_2^m$	$\alpha_1^m < \alpha_2^m$
$G_{b\alpha_2^m}^2 > -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$\tilde{W}_{b\alpha_2^m}^1 > 0$	$\tilde{W}_{b\alpha_2^m}^1 = 0$	$\tilde{W}_{b\alpha_2^m}^1 < 0$
$G_{b\alpha_2^m}^2 = -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$\tilde{W}_{b\alpha_2^m}^1 = 0$	$\tilde{W}_{b\alpha_2^m}^1 = 0$	$\tilde{W}_{b\alpha_2^m}^1 = 0$
$G_{b\alpha_2^m}^2 < -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$\tilde{W}_{b\alpha_2^m}^1 < 0$	$\tilde{W}_{b\alpha_2^m}^1 = 0$	$\tilde{W}_{b\alpha_2^m}^1 > 0$

Table 1: Signs of $\tilde{W}_{b\alpha_2^m}^1$ depending on α_1^m, α_2^m and $G_{b\alpha_2^m}^2$.

sign $W_{b\alpha_2^m}^1$ and the fact that $\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) = 0$ for $\alpha_1^m = \alpha_2^m$ lead to table 2.

	$\alpha_1^m > \alpha_2^m$	$\alpha_1^m = \alpha_2^m$	$\alpha_1^m < \alpha_2^m$
$G_{b\alpha_2^m}^2 > -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) < 0$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) = 0$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) < 0$
$G_{b\alpha_2^m}^2 = -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) = 0$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) = 0$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) = 0$
$G_{b\alpha_2^m}^2 < -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) > 0$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) = 0$	$\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m) > 0$

Table 2: Signs of $\tilde{W}_b^1(0, \alpha_1^m, \alpha_2^m)$ depending on α_1^m, α_2^m and $G_{b\alpha_2^m}^2$.

Finally, (21) enables us to derive table 3.

	$\alpha_1^m > \alpha_2^m$ Gov. 2 right-wing	$\alpha_1^m = \alpha_2^m$ Gov. 2 moderate	$\alpha_1^m < \alpha_2^m$ Gov. 2 left-wing
$G_{b\alpha_2^m}^2 > -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$b^m < 0$	$b^m = 0$	$b^m < 0$
$G_{b\alpha_2^m}^2 = -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$b^m = 0$	$b^m = 0$	$b^m = 0$
$G_{b\alpha_2^m}^2 < -\frac{U^2}{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2$	$b^m > 0$	$b^m = 0$	$b^m > 0$

Table 3: Signs of b^m depending on α_1^m, α_2^m and $G_{b\alpha_2^m}^2$.

Throughout the paper the importance of the cross derivative $G_{b\alpha_2^m}^2$ for determining the sign of b^m has been stressed. $G_{b\alpha_2^m}^2$ shows how heavily public consumption in the second period reacts to a change in b when various values of α_2^m are given.

First, it is to be reminded that $G_b^2 < 0$ (eq. 10b).

$G_{b\alpha_2^m}^2 > 0$ means that an infinitesimal increase (reduction) of b causes a smaller reduction (increase) of g_2 , if government 2 is left-wing. Alternatively, the more right-wing government 2 is, the larger is the increase (reduction) of g_2 , when b increases (decreases). In case of $G_{b\alpha_2^m}^2 < 0$ an infinitesimal increase (decrease) of b leads to a higher reduction (increase) of g_2 , if government 2 is left-wing. Conversely, the more right-wing government 2 is, the smaller is the reduction (increase) of g_2 .

Next, I analyze certain alternatives and try to give an interpretation. Throughout the following analysis it is assumed that the first period government is moderate, e. g. without extreme preferences for either the private or the public sector.

Case (i): $G_{b\alpha_2^m}^2 > 0$

(23) shows, that $G_{b\alpha_2^m}^2 > 0$ is sufficient for $G_{b\alpha_2^m}^2 > -\frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$. Thus the first line of table

3 is to be examined.

A government being sure to be reappointed at the beginning of the second period, leaves a balanced budget and a surplus otherwise.

Interpretation:

The first period government wants to buy insurance for its preferred size of the public sector, as it knows that a right-wing successor opts for a smaller public sector. But it also recognizes that the public sector in the second period increases in case of a decreasing b . Due to $G_{b\alpha_2^m}^2 > 0$ the increase of the public sector is larger compared to a situation where the first period government remains in office.

If the government will be replaced by a left-wing government it nevertheless leaves a surplus. The public sector in period 2 will be larger in comparison to the case $b = 0$. But this increase will be smaller than under a two-period-reign of government 1.

Case (ii): $G_{b\alpha_2^m}^2 = 0$

Here, government 2 always reacts the same to a change in b independent of its preferences.

Hence, $G_b^2(\alpha_2^{m'}, b) = G_b^2(\alpha_2^{m''}, b) \quad \forall b \wedge \forall \alpha_2^{m'}, \alpha_2^{m''} > 0. \quad G_{b\alpha_2^m}^2 = 0 \quad \text{implies}$

$G_{b\alpha_2^m}^2 > -\frac{U_{g_2 g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$, leading to the first line of table 3, we discussed in case (i).

Interpretation:

Anticipating a defeat in the next period's vote government 1 leaves a surplus. For any given value $b < 0$ government 2 promotes the public sector in the same way as government would do. Cases (i) and (ii) are the only alternatives allowing to derive a clear relationship between $\text{sign } G_{b\alpha_2^m}^2$ and $\text{sign } b$.

Case (iii): $G_{b\alpha_2^m}^2 < 0$

Now the behaviour of government 2 is contrary to case (i). The more right-wing government 2 is, the smaller is the influence b has on g_2 . As shown in table 3, we are not able to derive a clear sign of b , as this depends on the value of $G_{b\alpha_2^m}^2$ relative to the value of

$$-\frac{U_{g_2 g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}} < 0 .$$

Case (iii, a): $-\frac{U_{g_2 g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2} < G_{b\alpha_2^m}^2 < 0$

A government which is sure not to be reappointed leaves a surplus and a balanced budget otherwise.

$$\text{Case (iii, b): } G_{b\alpha_2^m}^2 = -\frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$$

Now the second line of table 3 is decisive. Government 1 always leaves a balanced budget and (10a) and (18b) show that

$$\text{sign}\{g_2^m - g_1^m\} = \text{sign}\{\gamma_2^m - \gamma_1^m\} = \text{sign}\{\omega_1^m - \omega_2^m\} = \text{sign}\{l_1 - l_2\} = \text{sign}\{\bar{x}_1 - x_2\} = \text{sign}\{\alpha_2^m - \alpha_1^m\}.$$

Although the budget will be always balanced, all political and private variables are not smoothed out, as long as government 1's preferences differ from those of government 2.

$$\text{Case (iii, c): } G_{b\alpha_2^m}^2 < -\frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$$

Here we deal with the third line of table 3. Facing a defeat in the next election government 1 realizes a deficit. Here a budget deficit can be explained by different preferences of succeeding governments.

4. Conclusions

The most important conclusion is that unbalanced budgets are a result of different political preferences between changing governments. A Stackelberg game occurs. But different preferences are not sufficient for the existence of unbalanced budgets or even deficits. Additionally it had to be examined how strongly government 2 reacts on a change in b .

Summing up the various cases we are able to derive the following results:

- 1) A government which is sure of being reappointed leaves a balanced budget and smoothes out all political variables.
- 2) If a right-wing (left-wing) government responds stronger (weaker) to a small change in b than a moderate government, the moderate government leaves a surplus.
- 3) If the reaction to a change in b is independent of government 2's political preferences, the moderate government leaves a surplus.
- 4) If a right-wing (left-wing) government responds weaker (stronger) to a small change in b than a moderate government, the sign of b is ambiguous. The exact value of depends on the value of $G_{b\alpha_2^m}^2$ relative to a certain threshold value (see (23)).

The appendix shows the sign of [...] in (23) (as well as the sign of b) depending on the properties of $U(g_2)$ and $H^2(l_2)$ only. G_b^2 , $G_{\alpha_2^m}^2$ and $G_{b\alpha_2^m}^2$ can be rewritten by using U_{g_2} , $U_{g_2g_2}$ and $U_{g_2g_2g_2}$ as well as l_2 , $H_{l_2}^2$, $H_{l_2l_2}^2$, $H_{l_2l_2l_2}^2$ and $H_{l_2l_2l_2l_2}^2$. A similar context could be found in the model of Tabellini and Alesina (1990). In this model the sign of b depends heavily on the first derivative $\lambda_k(k) = -\frac{U_{kkk}(k)U_k(k) - 2[U_{kk}(k)]^2}{[U_k(k)]^3}$ of the so-called ‘concavity index’,¹⁰ $\lambda(k) = -\frac{U_{kk}}{[U_k]^2} > 0$ of the utility function $U(k)$ (Tabellini and Alesina, 1990, p. 42).

Table 4 compares the conditions for the sign of b in the two models.

	Tabellini and Alesina	Current model
$b^m < 0$	$\alpha_1^m \neq \alpha_2^m \wedge$ $\lambda_k(k) = -\frac{U_{kkk}(k)U_k(k) - 2[U_{kk}(k)]^2}{[U_k(k)]^3} > 0$	$\alpha_1^m \neq \alpha_2^m \wedge$ $G_{b\alpha_2^m}^2 > -\frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$
$b^m = 0$	$\alpha_1^m = \alpha_2^m \vee$ $\lambda_k(k) = -\frac{U_{kkk}(k)U_k(k) - 2[U_{kk}(k)]^2}{[U_k(k)]^3} = 0$	$\alpha_1^m = \alpha_2^m \vee$ $G_{b\alpha_2^m}^2 = -\frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$
$b^m > 0$	$\alpha_1^m \neq \alpha_2^m \wedge$ $\lambda_k(k) = -\frac{U_{kkk}(k)U_k(k) - 2[U_{kk}(k)]^2}{[U_k(k)]^3} < 0$	$\alpha_1^m \neq \alpha_2^m \wedge$ $G_{b\alpha_2^m}^2 < -\frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$

Table 4: Conditions for $sign b^m$ in Tabellini and Alesina (1990) and the current model.

It can be supposed that a coherence between $sign \lambda_k(k)$ and $sign \left\{ G_{b\alpha_2^m}^2 + \frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2} \right\}$

may exist. In order to obtain a partial result, I use the simple functions $H^2(l_2) = -\frac{1}{2}l_2^2 + 2l_2$

and $U(g_2) = -\frac{1}{2}g_2^2 + 4g_2$. The appendix shows that $U_{g_2g_2} \cdot G_{\alpha_2^m}^2 \cdot G_b^2 + U_{g_2} \cdot G_{b\alpha_2^m}^2 < 0$ for

¹⁰ In the original paper from Debreu and Koopmans (1982), $\lambda(k)$ is called the ‘convexity index’.

$l_2 \in \left] \frac{1}{2}, 1 \right]$ and $g_2 \in [0, 2]$ (Figure 1)¹¹. This implies $G_{ba_2^m}^2 < -\frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2}$ (see (23)). Any

interior solution of (83) will yield $b^m > 0$ if $\alpha_1^m \neq \alpha_2^m$.

Due to $U_{g_2}^2 > 0, U_{g_2g_2}^2 < 0$ and $U_{g_2g_2g_2}^2 = 0$ we have $\lambda_{g_2}(g_2) > 0$ and this is a contradiction to

the former conjecture about the coherence of $\text{sign } \lambda_k(k)$ and $\text{sign} \left\{ G_{ba_2^m}^2 + \frac{U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2}{U_{g_2}^2} \right\}$.

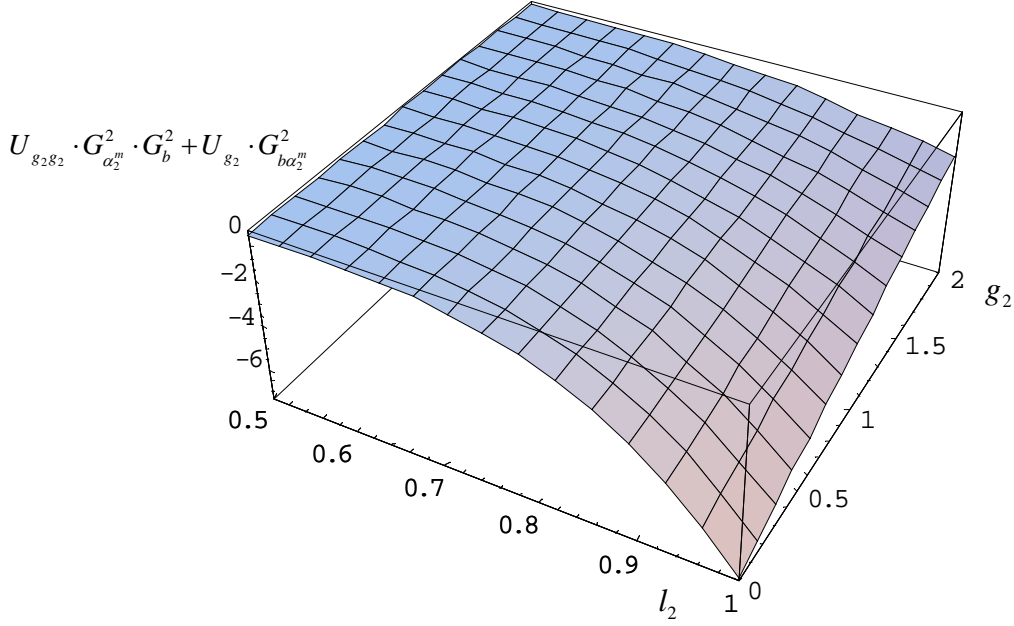


Figure 1: $U_{g_2g_2}^2 \cdot G_{\alpha_2^m}^2 \cdot G_b^2 + U_{g_2}^2 \cdot G_{ba_2^m}^2$ if $l_2 \in \left] \frac{1}{2}, 1 \right]$ and $g_2 \in [0, 2]$ ¹².

However, it has to be stressed that the value of $G_{ba_2^m}^2$ hinges on the fourth derivative $H_{l_2l_2l_2l_2}^2$, which is $H_{l_2l_2l_2l_2}^2 = 0$ in my example. On the other hand, $\lambda_k(k)$ comprises the first three derivatives of the partial utility functions only. Furthermore I assumed utility to be linear in second period's private consumption.

[to be continued]

¹¹ The limitation $l_2 \in \left] \frac{1}{2}, 1 \right]$ ensures that $\Omega_{r_2}^2 = -\frac{1}{2l_2 - 1} < 0$ and $\Omega_{r_2r_2}^2 = -\frac{2}{(2l_2 - 1)^3} < 0$ according to (A2)

and (A3).

¹² Figure 1 and the affiliated calculations were designed by using *MATHEMATICA 3.0*.

Appendix

1. Determination of $G_{b\alpha_2^m}^2$

The cross derivative $G_{b\alpha_2^m}^2$ from (22) is

$$G_{b\alpha_2^m}^2 = \frac{\left[U_{g_2 g_2} + \alpha_2^m U_{g_2 g_2 g_2} G_{\alpha_2^m}^2 \right] \cdot \left\{ \Omega_{\gamma_2 \gamma_2}^2 l_2 + \left(\Omega_{\gamma_2}^2 \right)^2 L_{\omega_2}^2 \right\} - \alpha_2^m U_{g_2 g_2} G_{\alpha_2^m}^2 \left\{ \Omega_{\gamma_2 \gamma_2 \gamma_2}^2 l_2 + \Omega_{\gamma_2}^2 \left[3L_{\omega_2}^2 \Omega_{\gamma_2 \gamma_2}^2 + L_{\omega_2 \omega_2}^2 \left(\Omega_{\gamma_2}^2 \right)^2 \right] \right\}}{\left\{ \Omega_{\gamma_2 \gamma_2}^2 l_2 + \left(\Omega_{\gamma_2}^2 \right)^2 L_{\omega_2}^2 + \alpha_2^m U_{g_2 g_2} \right\}^2}.$$

We want to show that $G_{b\alpha_2^m}^2$ depends on the properties of $U(g_2)$ and $H^2(l_2)$ only. First we

have $L_{\omega_2}^2 = -\frac{1}{H_{l_2 l_2}^2}$ (eq. 7). Differentiating (7) with respect to ω_2 yields

$$(A1) \quad L_{\omega_2 \omega_2}^2 = -\frac{H_{l_2 l_2 l_2}^2}{\left(H_{l_2 l_2}^2 \right)^3},$$

and invoking (7) in (8) yields

$$(A2) \quad \Omega_{\gamma_2}^2 = -\frac{1}{\left(1 + H_{l_2}^2 \right) \cdot \frac{1}{H_{l_2 l_2}^2} + l_2} < 0.$$

(A2) is differentiated twice to¹³

$$(A3) \quad \Omega_{\gamma_2 \gamma_2}^2 = \frac{2L_{\omega_2}^2 - (1 - \omega_2)L_{\omega_2 \omega_2}^2}{\left[(1 - \omega_2)L_{\omega_2}^2 - l_2 \right]^3} = \frac{\left(1 + H_{l_2}^2 \right) \frac{H_{l_2 l_2 l_2}^2}{\left(H_{l_2 l_2}^2 \right)^3} - \frac{2}{H_{l_2 l_2}^2}}{\left[-\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} - l_2 \right]^3} < 0$$

and

$$(A4) \quad \Omega_{\gamma_2 \gamma_2 \gamma_2}^2 = \frac{-12 \left(H_{l_2 l_2}^2 \right)^3 - 3 \left(H_{l_2 l_2}^2 \right)^2 H_{l_2 l_2 l_2}^2 l_2 - \left(1 + H_{l_2}^2 \right) \cdot Q(l_2)}{\left(1 + H_{l_2}^2 + H_{l_2 l_2}^2 l_2 \right)^5},$$

and it is defined: $Q(l_2) := \left(1 + H_{l_2}^2 \right) H_{l_2 l_2 l_2}^2 - 3 \left(H_{l_2 l_2}^2 \right)^2 l_2 - H_{l_2}^2 \cdot \left(9 H_{l_2 l_2}^2 - H_{l_2 l_2 l_2}^2 l_2 \right)$.

Next, we want (9) to be transformed into $\alpha_2^m = -\frac{\Omega_{\gamma_2}^2 \cdot l_2}{U_{g_2}}$ for $\alpha_2^j = \alpha_2^m$. Regarding (A2) we

make another transformation into

$$(A5) \quad \alpha_2^m = \frac{l_2}{U_{g_2} \cdot \left[\left(1 + H_{l_2}^2 \right) \cdot \frac{1}{H_{l_2 l_2}^2} + l_2 \right]}.$$

¹³ The negative signs of (A2) and (A3) are owed to (15b) and (23).

Finally, $G_{\alpha_2^m}^2$ is to be analyzed. (7), (A2), (A3) and (A5) are inserted into (10a), leading to

$$(A6) \quad G_{\alpha_2^m}^2 = \frac{U_{g_2}}{\frac{\left(1 + H_{l_2}^2\right) \frac{H_{l_2 l_2}^2}{\left(H_{l_2 l_2}^2\right)^3} - \frac{2}{H_{l_2 l_2}^2}}{\left[\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} + l_2\right]^3} \cdot l_2 + \left(\frac{1}{\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} + l_2}\right)^2 \cdot \frac{1}{H_{l_2 l_2}^2} - \frac{U_{g_2 g_2} \cdot l_2}{U_{g_2} \cdot \left[\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} + l_2\right]}}$$

Hence $G_{\alpha_2^m}^2$ can be expressed by means of U_{g_2} , $U_{g_2 g_2}$ and $U_{g_2 g_2 g_2}$ as well as

l_2 , $H_{l_2}^2$, $H_{l_2 l_2}^2$, $H_{l_2 l_2 l_2}^2$ and $H_{l_2 l_2 l_2 l_2}^2$.

2. Determination of G_b^2

According to (10b) G_b^2 is described by $-\frac{\Omega_{\gamma_2 \gamma_2}^2 \cdot l_2 + \left(\Omega_{\gamma_2}^2\right)^2 \cdot L_{\omega_2}^2}{\Omega_{\gamma_2 \gamma_2}^2 \cdot l_2 + \left(\Omega_{\gamma_2}^2\right)^2 \cdot L_{\omega_2}^2 + \alpha_2^m U_{g_2 g_2}}$. Using (7), (A2),

(A3) and (A5) we finally have

$$(A7) \quad G_b^2 = \frac{\frac{\left(1 + H_{l_2}^2\right) \frac{H_{l_2 l_2}^2}{\left(H_{l_2 l_2}^2\right)^3} - \frac{2}{H_{l_2 l_2}^2}}{\left[-\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} - l_2\right]^3} \cdot l_2 - \left(\frac{1}{\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} + l_2}\right)^2 \cdot \frac{1}{H_{l_2 l_2}^2}}{\frac{\left(1 + H_{l_2}^2\right) \frac{H_{l_2 l_2}^2}{\left(H_{l_2 l_2}^2\right)^3} - \frac{2}{H_{l_2 l_2}^2}}{\left[\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} + l_2\right]^3} \cdot l_2 + \left(\frac{1}{\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} + l_2}\right)^2 \cdot \frac{1}{H_{l_2 l_2}^2} - \frac{U_{g_2 g_2} \cdot l_2}{U_{g_2} \cdot \left[\frac{1 + H_{l_2}^2}{H_{l_2 l_2}^2} + l_2\right]}}$$

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