

The Dark Side of Political Competition[†]

An Application of Conflict Models to Politics

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Abstract

Further refining Anthony Downs' economic theory of politics, competition for popularity is modeled here as an industry in which political factions are represented by principal-agent networks engaged in campaigns of attrition aimed at the supporters of the opposition. It is shown that the outcome of attritional political competition depends on the available mechanisms of interaction, which are represented here by different versions of the Chase-Osipov-Lanchester conflict model. We show that interaction mechanisms influence the chances to win a majority vote and significantly expand the notion of agenda-setting power introducing control structures over procedures. The model can be tested and applied to intra-party campaigns, fact-finding committees or juries.

[†]Work in progress. Comments are welcome. Please do not quote. The author thanks Ruediger Pethig for valuable comment.

1 Introduction

The only force that can overcome an idea and a faith is another and better idea and faith, positively and fearlessly upheld.

DOROTHY THOMPSON (1893–1961)

In this paper we will introduce a *dynamic theory of campaigning* aimed at expanding the domain economics to cover those forms of political competition based on aggressive rivalry. We will treat politics in this context as an industry in which firms—networks of agents—compete by directly attempting to disable their opponents (Hirshleifer 1985, 64). One particularly interesting result is that control rights over how a debate is organized and how long it takes play a crucial role in determining the outcome of a vote and will therefore be taken into account by political entrepreneurs. The theory presented here is thus a modification of mainstream public choice. What it retains is the fundamental assumption of competition for popularity. What it modifies is the way competition is modelled. What it adds is the dimension of time.

Indeed, political competition has a dark side and our technical terms in the field of politics reveal the close connection between politics and fighting. The word congress for instance is derived from the Roman word for combat, *congressus*. The denomination *commission* is derived from Roman military jargon, too, and refers to an official document issued by a government conferring on the recipient the rank of an officer in the armed forces. In the German tradition of consensual decision-making a vote on a highly controversial issue with a close margin is literally called a ‘combat-vote’ (*Kampfabstimmung*). The word *campaign* originally marked an open country suited to military maneuvers.

Campaigning to win a vote is consequently modelled here as an attrition-based conflict similar to that between armies on a battlefield. This approach may at first sight seem odd, but indeed on closer look one recognizes many structural similarities between pitched battle and democratic procedures: opposing groups meet in a certain place at a certain time in order to take a decision by using formal voting methods where typically one vote eliminates another and majorities win. Minorities will only be able to win a majority vote if each voter can somehow eliminate more opposed votes than the one she could eliminate with her own. This is one of the reasons why despite existing coalitions and partisan discipline debates still take place in politics.

As there are many forms of combat there are many forms of debate, ranging from plenary sessions to confidential talks. What is needed to cover those

forms is an *economic theory of political interaction*. Depending on the circumstances winning a vote may in the end boil down to being more effective in debate, and often effectiveness depends from *how* we debate. Speaking of *debate* the word is derived from 13th century French and originally meant ‘to beat down completely’. How one party ‘beats down’ another, and the importance of rules and procedures in this context shall be analyzed in the following sections.

2 A Model for the Dark Side of Politics

Ever since Anthony Downs introduced the idea of competing political entrepreneurs the view was held that political competition would be more or less analogous to *inter-firm competition* (Downs 1957). In a two party world with complete information and unimodal symmetric voter preference profiles each party’s optimal strategy would be to make the election pledges maximizing the median voter’s welfare. In Downs’ world the political entrepreneurs are the principals of firms supplying election pledges. The voters, as their main consumers, reveal their preferences for the party programmes by going to the ballots. Elaborating on Downs’ close analogy between inter-firm and inter-party competition Tullock annexed the shrewd question: “What is the difference between a political party and a department store?” (Tullock 1965, 458)

While the Downsian paradigm of inter-firm competition covers many important aspects it still appears elusive in some cases, so for instance if there is no clear distinction between parties and voters. Indeed, in many cases like intra-party debates, fact finding committees or juries the factions in competition are formed by the very voters. Besides, when voters are influenceable campaigning may result in *preference-shaping*. For the political entrepreneur, then, competition tends to be of the ruinous type, resembling a battlefield rather than a marketplace.

We shall deal here with that aspect of political competition in which the firms themselves are exposed to competitive efforts of the other side, that is to say politics when it takes the form of *pure conflict*. A seasoned analytic workhorse for modelling such conflict is the Chase-Osipov-Lanchester ordinary differential equation model (henceforth abbreviated COL-model) which is used to predict the time dependent state of a battle.¹ It seems the same

¹ The model is better known as the Lanchester model, after British aeronautics pioneer Frederick William Lanchester (1868-1945). He however seems to have been precurred

reasons for why the COL-model is considered inadequate for modelling modern combat, like the absence of force movements, spatial and terrain effects, make it ideal for modelling political competition processes where two groups are trying to influence each other's members. The *microtechnology of conflict* affects how the committed forces of each side enter into determining the outcome of the battle (Hirshleifer 2000, 782).

From an economics perspective “the central element in the conduct of battle is essentially not an actual engagement of opposing forces, but rather a confrontation of two opposing *networks* of agents, in which it may be that network rather than the agents themselves which may be the object of destruction” (Brennan/Tullock 1982, 232). Modelling political parties as networks consisting of a political entrepreneur and the voters we can apply the COL-model to a political competition process. If the network itself, and thus the outcome of a vote, can be manipulated by exerting an effort aimed at eliminating opposed political agents—for instance by convincing them to change sides or to abstain from a vote—the principals are in conflict with each other. The outcome will then at least partially depend from the microtechnology of political competition or, put differently, the *mechanism of interaction* between the networks engaged in conflict.

Our point of view is that things like plenary debates, meetings in subcommittees, informal conversations, small-talk in the lobbies or even dialogues in confidence are such mechanisms of interaction, and that they are not equivalent regarding their influence on the chances to win a vote. A rational political entrepreneur will accordingly ask herself how to make sure the microtechnology favouring the own party will be used, and, if possible, try to acquire the right to exclude others from choosing between available technologies. The hypothesis is that exploiting parliamentary procedures opens the way to exclusive control.

In Section 3.2 we will make some assumptions about the principal-agent networks and the interaction between individual agents. In Section 3.5 we will use the COL-model to analyze the interaction process between the networks. We derive the proposition that in some cases gaining control over interaction processes may determine the winner in Section 4. Some practices typically observed in partisan politics can be explained by our findings.

by two unanvowed scientists, American Navy Lieutenant J.V. Chase and M. Osipov from Russia, so the correct name should be the Chase-Osipov-Lanchester ordinary differential equation model.

3 Political Interaction Mechanisms

We will confine the model to two principal-agent networks, x and y , each consisting of a number of individual agents which are in disagreement with each other about a collective decision. Both are led by a Downsian political entrepreneur, who acts as an opinion-leader and provides incentives for her agents to exert an effort aimed at eroding the support for the opposed principal.

The numeric strength of the two factions at every point in time $t \in \mathbb{R}^+$ shall be $x(t)$ and $y(t)$ with $x, y \in \mathbb{Q}^+$.² They compete with each other for winning a vote over their respective motions. What we have here is thus not competition by politicians for votes but competition by voters for votes.

Think about the two groups as parties or factions in a collective decision-making body like a committee. Factions of this kind can be found in *intra-party campaigns* where members do not necessarily vote along partisan lines. Typically such campaigns are for leadership positions, about questions of principle or motions regarding highly controversial but complicated topics where the rational ignorance hypothesis holds, and usually take place during party congresses where many contacts take place in the lobbies, during breaks etc. Interaction in *fact-finding committees* or boards of inquiry are another application with a very interesting case being that of *juries* (Pitsoulis 2006).

3.1 Partisan Discipline and Rational Ignorance

We assume the contracts between the principal and the party members to define *partisan discipline* in the following way: each agent is bound both to support the principal's cause by actively persuading agents of the other party as well as to vote for the proposal. Partisan discipline however may not prevent the agents to keep out of the discussion *and* the vote if having been convinced by the other side. There are thus two exclusive modes of behavior for the agents. In *active* mode an agent i both exerts an effort e_i to convince opposed agents and will vote for her principal's motion. Think of the efforts as the production of verbal or written statements aimed at convincing opposed supporters. In *passive* mode she will neither exert any effort to convince others nor vote for her principal's motion.

The effectiveness of an agent in convincing opponents to switch to the passive mode is assumed to be dependent solely on the effort, which is determined

² This is just a technical simplification. Of course the number of members of a group must be a discrete integer.

here both by incentives derived from the contract with the principal and intra-party social pressure. The reason is that the voter-supporters are assumed to be rationally ignorant about the issue which makes them *influenceable*. The influencing of agents takes place each time they interact. Interaction can take place in a plenary debate, confidential talks etc. An agent facing an effective attempt of persuasion is for reasons of simplicity assumed to immediately switch to and remain in passive mode. Underlying is of course the assumption that the communication of efforts between two agents is effective, which rules out transmission losses and misunderstandings.

Some form of interaction lead to contests between agents concentrating their effort with the strategic aim to convince other agents. Party interaction is a typical *team production* problem (Alchian/Demsetz 1972, Holmström 1982). In a plenary debate the number of active agents might be verifiable by the principal, but other interaction mechanisms do not reveal the level of active support before the vote. Usually the vote will take place only after some time and thus one agent's efforts and her contribution to the team output will not be observable.

3.2 A Simple Microfoundation

The approach roughly follows Kandel/Lazear (1992). Assume the team production function of the $i = 1, \dots, n$ political agents in the party x to be of the form

$$X = \sum_{i=1}^n e_i + s \prod_{i=1}^n e_i \quad (1)$$

where $0 < s < 1$ is the weight of the economies of scope from joint production. The team outputs X, Y play a secondary role here and serve mainly as a basis to calculate the payoffs y_i to the agents and for deriving the optimal efforts which will later determine the effectiveness in convincing agents of the opposed party to switch to passive mode. Assume here each agent is paid an equal profit share $y_i = x/n$ of the team product. Note that the team output is not just a purely abstract term: it represents the accounting records of the party's efforts—the minutes of proceedings so to say—containing all verbal or written statements.

Efforts burdens the agent with a cost $C(e_i)$. The cost function has the properties $\partial C/\partial e_i > 0$ and $\partial^2 C/\partial e_i^2 > 0$. It shall for the sake of simplicity be assumed here to be

$$C(e_i) = \frac{1}{2} e_i^2. \quad (2)$$

The agent i 's net utility shall be

$$u_i = y_i - C(e_i) \quad (3)$$

where y_i denotes income paid by the principal.

If monitoring and enforcement of the agents' efforts would be feasible and costless the principal would maximize the profit Π of the party over the individual efforts. From

$$\max_{e_i} \Pi = X - \sum_{i=1}^n C(e_i) \quad (4)$$

for $i = 1, \dots, n$ the first order conditions

$$\frac{\partial \Pi}{\partial e_i} = 1 + s \prod_{j \neq i} e_j - e_i = 0 \quad (5)$$

can be derived, leading to the socially optimal effort e_i^* and a first best utility u_i^* . For the simplest possible case of two agents follows

$$e_i^* = \frac{1}{1-s}. \quad (6)$$

If however the individual effort of the representative agent can not be verified incentives to enjoy a free ride will enter the agent's decision. Agent i will then maximize her individual net utility and derive from the system of first order conditions

$$\frac{\partial u_i}{\partial e_i} = \frac{1}{n} \left(1 + s \prod_{j \neq i} e_j \right) - e_i = 0 \quad (7)$$

the Nash equilibrium effort $e_i^N < e_i^*$ which clearly leads to a utility $u_i^N < u_i^*$. For the exemplary case of two agents we get

$$e_i^N = \frac{1}{2-s}. \quad (8)$$

It shall be further assumed that the moral hazard of the team members is held in check by *social pressure* among the agents. Seen from the viewpoint of a principal peer pressure is a good providing an additional incentive; for the agents peer pressure is a (necessary) bad. As an agent affected by peer pressure can be compensated with wealth for her welfare loss it can be translated into monetary terms and thus can be thought of being additional costs to the agent.

The peer pressure on agent i is given by

$$p_i = P(e_i, \mathbf{e}_j) \quad (9)$$

where e_i is the own effort and \mathbf{e}_j the vector of the $j \neq i$ other team members' observed efforts. A high own effort will ceteris paribus lead to less pressure applied by the peers while a high effort of the others will result in higher pressure so that $\partial P / \partial e_i < 0$ and $\partial P / e_j > 0$ for some j . Let us simply assume here

$$p_i = k(\max(e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n) - e_i) \quad (10)$$

with the constant weight $k > 0$ measuring the influence peer pressure has on agent i 's utility

$$u_i = y_i - C(e_i) - p_i. \quad (11)$$

The system of first order conditions

$$\frac{\partial u_i}{\partial e_i} = \frac{1}{n} \left(1 + s \prod_{j \neq i} e_j \right) - e_i + k = 0 \quad (12)$$

can be solved for the optimal effort e_i^P which inserted into the utility function leads to the optimal utility u_i^P under peer pressure. In the case of two agents we get

$$e_i^P = \frac{1 + 2k}{2 - s}. \quad (13)$$

This effort will enter the strategic interaction model as the *effectiveness* of the individual political agent in convincing agents of the other party to switch to the passive mode of behaviour. The more effective an agent is the more 'lethal' are her arguments.

We have assumed here neither peer pressure nor the incentive schemes offered by the principals are interdependent and therefore need not model the interaction between the two parties as a dynamic game. The representative agent will exert a *static* effort at each point in model time. What on first look appears as a shortcoming allows us to concentrate on analyzing the interaction between political agencies in the *short run*, where *rigidities of the incentive structures* apply.

3.3 Political Competition at Grassroots Level

Now let us approach the question how the individual effort enters into determining the outcome of interaction. When two agents meet one will try

to overcome the other and impose his own opinion on her. Interpreting the individual efforts as information uncovered interaction may shift the agents' preferences. Information might however be disinformation or everything else which destroys the principal-agent network of the opposed political entrepreneur.

We simplify this complex question and assume that the *contest success function*

$$\frac{c}{b} = \left(\frac{e_x^p}{e_y^p} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\omega}{1-\omega} \right)^{\frac{\beta}{1-\beta}} \quad (14)$$

describes how agents compare the quality of an argument brought forward, resulting in the relative probability of success, c/b , with $0 < b < 1$, $0 < c < 1$ and $b + c \equiv 1$.³ The parameter $0 < \omega < 1$ measures a degree of *general ethical incompatibility* which enables us to cover cases in which one side has for instance the morally or legally better position. α, β play a crucial role as they determine the degree to which a greater effort translates into a greater chance to convince an agent of switching to passive behaviour. α scales the degree of success of efforts and may be called the *influencability parameter*. It makes sense to assume $\alpha, \beta < 1$ so that neither a greater effort nor a greater degree of default does ceteris paribus lead to a greater success chance. Clearly, if efforts are equal the relative success chance will be 50:50.

In essence, what we have here is just the assumption that agents with stronger incentives will ceteris paribus have a greater success chance in convincing opponents. If interaction takes place between two agents one of them will be convinced to remain passive. We nevertheless still treat the process of how an agent forms her opinion, when confronted with an effort aimed at her ability or remain active, as a black box. An agent may change her opinion because new information is revealed to her, because the opinion-leader is successfully slandered or because she is being threatened. Be that as it may, what is important for the principal of a faction is not so much the opinion-formation process of the agents but the success chances to win a vote.

3.4 Voting and Procedures

Votes play the crucial role of revealing the level of support to the political entrepreneurs. Regarding the voting rules we assume that the principle 'one-agent-one-vote' holds. The outcome of a vote may well influence the agents'

³ This is similar to Hirshleifer (2000), 789.

utility, but let us assume here that the principal provides for sufficient compensation over vote-induced changes in the agents' reservation utility.

It is assumed here that at least one majority vote must be taken at some point in time. It is not necessary to make any special assumptions about which rule is used. The voting rule must only fulfill opting-out condition: abstentions by supporter-voters of one of the two factions are not added to the blocking minority and do therefore not influence the outcome. In our examples we will deal with opting-out single majority votes (henceforth abbreviated SMV) and opting-out unanimity votes (henceforth OUV).

A point of interest arises if we ask *when* the vote takes place. The timing of the vote is absolutely crucial if the willingness to vote can be eliminated by concentrated efforts. Procedures play a central role here as they can be used to acquire control over the timing of the vote and the interaction mechanism. We therefore introduce a control structure.

Definition 1: Let S be the set $\{x,y\}$ of the two political entrepreneurs and \tilde{S} a subset of S . The *procedure* A is defined as a set of rights $\{V,I\}$ where V denotes the right to fix the timing of the vote and I the right to determine the interaction mechanism. Let \tilde{A} denote a subset of A . Define $\alpha(\tilde{S})$ as the *control structure over the procedure*.

According to Definition 1 *exclusive control* over the timing of the vote and the interaction is given if the subset \tilde{S} contains only one principal and the control structure includes both right, i.e. $\alpha(\tilde{S}) = \{V,I\}$. In the following sections we will only deal with exclusive control structures. We will for now treat the allocation of control rights as exogenously given.

3.5 Campaigning: Two Microtechnologies of Interaction

We now use two versions of the COL-model to describe the attrition of support due to attritional campaigning (i.e. convincing members of a party to switch to passive mode) and different mechanisms of interaction between the parties. The first is one where interaction between individual agents is organized in such a way that a concentration of efforts has economies of scale in the attrition of support. This form of interaction covers 'aimed' attempts of persuading opposed agents, which may take place in a public debate or informal talks.

The second model of interaction covers those mechanisms of opinion exchange where attempts of persuasion are not directed at individuals but rather 'un-aimed' public statements which affect every agent reached. A typical case is

the public plenary debate. The model covers confidential talks between two agents as well, so actually the two models cover three different mechanisms.

3.5.1 Concentrated Interaction

Let us assume that a debate takes place and every agent of each party interacts once at each point in time. During interaction faction x experiences an (expected) loss rate of active supporters proportional to the number of opposed agents, $-by(t)$ while y expects to lose voice with the rate c for each member of x . We shall for simplicity assume that no side has a moral advantage and thus $\omega/(1-\omega) \equiv 1$.

Interaction then can be reduced to a linear homogenous system of differential equations⁴

$$\text{M1: } \begin{cases} \dot{x} = -by, \\ \dot{y} = -cx. \end{cases} \quad (15)$$

The support loss rate is proportional to the numerical strength of the opposed party. M1 is a microtechnology of *pure support attrition*—there are no active agents switching sides and no exogenous losses of active supporters. *Exit* from a party comes in other words by the denial of *voice* (Hirschman 1970).⁵

Note that M1 is equivalent to

$$\begin{aligned} cx\dot{x} - by\dot{y} &= 0 \\ \Leftrightarrow \frac{dx}{dy} &= \frac{by}{cx}. \end{aligned} \quad (16)$$

It can be analyzed in a phase diagram. By separating the variables and integrating from (x_0, y_0) to $(x(t), y(t))$ we obtain the solution for the initial-value problem $x(0) \equiv x_0$ and $y(0) \equiv y_0$

$$b \int_{y_0}^{y(t)} y dy = c \int_{x_0}^{x(t)} x dx \quad (17)$$

from which follows

$$b(y^2(t) - y_0^2) = c(x^2(t) - x_0^2). \quad (18)$$

This quadratic relationship between x and y is the reason why the linear system M1 has been given the name “square law model” (Coleman 1982,

⁴ We closely follow the concise summary given in Coleman (1982), 113–5.

⁵ In an application of the model to interaction within juries exit from a faction in the form of terminating the principal-agent contract is included in the differential equation system. See Pitsoulis (2006).

113). From simple transformations of (18) we obtain the state equations for each party's numeric strength

$$y(t) = \sqrt{y_0^2 + \frac{c}{b}(x^2(t) - x_0^2)} \quad (19)$$

and

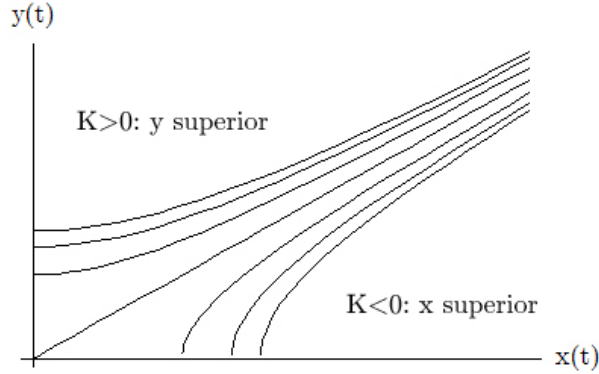
$$x(t) = \sqrt{x_0^2 + \frac{b}{c}(y^2(t) - y_0^2)}. \quad (20)$$

Now define $by_0^2 - cx_0^2 \equiv K$ and rewrite (18) as

$$K = by^2 - cx^2. \quad (21)$$

The graph of (21) is a hyperbola for $K \neq 0$ but a straight line for $K = 0$. All points of $K \in \mathbb{R} \rightarrow \mathbb{R}^2$ belong to an orbit. Figure 1 shows orbits for exogenously given different initial-value combinations (x_0, y_0) and given b, c .

Fig. 1: Support Attrition Trajectories for (21)



Obviously one party will end up having no active supporters left if the interaction continues long enough. The reason is that the opponents are superior in political competition: because of greater numbers, higher effectiveness or a combination of both one of the two teams is more successful in convincing opponents to switch to passive behaviour. Let us now define the condition for being more successful in political campaigning.

Definition 2: Let $b, c, x_0, y_0 > 0$ and (x^*, y^*) the point of crossing of the solution's orbit with an axis of the positive quadrant of the phase diagram at time T . If the point of crossing fulfills the condition $y^* = 0$ and $x^* > 0$ the x -party is *superior* and vice versa. If $x^* = y^* = 0$ a *stalemate* occurs.

Being superior or inferior in campaigning does have important consequences for the optimal timing of the vote. If $K > 0$ the y -party is superior and the orbit crosses the y -axis at $(0, \sqrt{K/b})$. If $K < 0$ the party x is superior and the orbit crosses the x -axis at $(\sqrt{-K/c}, 0)$. A superior party does either have an initial numerical superiority or is sufficiently effective to become a majority despite initially being a minority. In any case it will be able to win a vote *if* the principal has exclusive control over timing of the vote, i.e. the control set α contains V and the opponent is excluded.

A stalemate occurs if and only if

$$\frac{y_0}{x_0} = \sqrt{\frac{c}{b}} \quad (22)$$

meaning that in order to achieve a stalemate an opposing faction twice as numerous one must be four times as effective in convincing opponents not to vote. There is a special case of a stalemate to be considered.

Definition 3: Parties are *symmetric* if they are identical both in initial values x_0, y_0 and effectiveness coefficients b, c .

Symmetry implies equally ruinous political competition. This intuition directly leads a first result.

Proposition 1: *Symmetry results in an impossibility to take a collective decision by majority voting rules.*

Proof: The proof is trivial. If both factions have exactly the same initial numerical strength as well as effectiveness coefficients, and if no party has an ethical or moral superior position, then $x(t)$ is equal to $y(t)$ for all t . ■

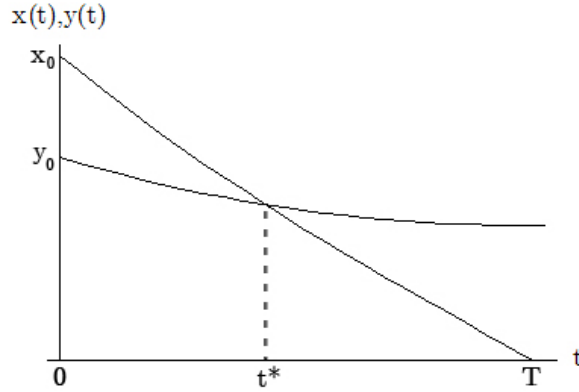
For any party to win the relative effectiveness must exceed the square of the initial relative strength. Mechanisms working in the fashion of M1 therefore confer a decisive advantage to be gained from numerical strength which directly translates into a higher loss of support for the other side. A superior party profits from the fact that each active agent is capable of taking part in the exchange of opinions with a disproportionately decreasing number of active opponents.

If the incentive structure is rigid the (not necessarily optimal) efforts b, c can not be manipulated. The initial values x_0, y_0 are exogenous. Political competition is therefore shifted to the field of control over the *procedures*.

In order to demonstrate this point we have to find the general solutions for $x(t)$ and $y(t)$ which each consist of the sum of the two linearly independent solutions of M1 and which can be written either as exponentials or as hyperbolic sines and cosines.⁶

A graph for an exemplary case with $K > 0$ is shown in Figure 2.⁷ It shows that although numerical superiority has a quadratic effect the larger party may still loose a vote if the opposing political agents are effective enough and can stretch the campaign over a long enough time.

Fig. 2: General Solutions of (15) for some $K > 0$



From time $t = 0$ until t^* party x would win a SMV because $x(t) > y(t)$, while y would win such a vote from t^* until T because $y(t) > x(t)$. Generally, if y is superior a total of $\sqrt{K/b}$ of its agents will remain active when eventually in

$$T = \frac{1}{2\sqrt{bc}} \left(\ln \left(\frac{\sqrt{b}}{\sqrt{c}} y_0 + x_0 \right) \right) - \left(\ln \left(\frac{\sqrt{b}}{\sqrt{c}} y_0 - x_0 \right) \right) \quad (23)$$

the last member of x has been convinced to remain passive. Accordingly, at each point in time after T party y would win an OUV.⁸ The opposite holds for the case of x being superior. Under such circumstances not only winning a vote clearly depends on *when* the vote is taken. This observation leads us to the next proposition.

⁶ For the derivation of the general solutions see Appendix A.1.

⁷ The values in this example are $x_0 = 30$, $y_0 = 20$, $b = 0.4$ and $c = 0.1$.

⁸ Working with opting-out rules is a simplification here, as the aim is to concentrate on pure attrition of support. Opting-out rules, where abstentions are not distortionary are the most common form of voting rules.

Proposition 2: *If and only if the parties are symmetric the control structure over the timing of the vote does not determine the winner of an opting-out vote.*

Sketch of Proof: Consider the possible cases. If one party is superior (a) either the superior one remains numerically larger from $t = 0$ until T , (b) both parties are equally large in $t = 0$ or (c) there will be a point in time t^* where numerical superiority is being lost. This case arises if the superior party has started with a lower initial value than the inferior party. In case (a) the allocation of control rights over the timing of the vote will determine under which voting rule the superior party will win: from $t = 0$ until T it would win (at least) a SMV vote while it would win an OUV at each point in time after T . In case (b) there will either be a draw in $t = 0$ or a definitive winner. In case (c) the allocation of control rights over the timing of the vote are crucial, as from $t = 0$ until t^* the initially larger but inferior party would win (at least) under the SMV rule, while from t^* until T the initially smaller but superior party would win under the same rule. After T it would win a vote under the OUV rule. A stalemate can take place either with (d) one party being initially larger or (e) both being equally numerous until T . In case (d) the allocation of control rights will determine the winner at different t , as above. In case of symmetry (e) there is not only a numerical draw from $t = 0$ until T but the effectiveness coefficients must be equal, so this is the only case where the allocation of control rights over the timing of the vote does not change the outcome under any voting rule. ■

Although trivial it may be worth pointing out to the fact that if the principal of a superior party has the control right over the timing of the vote she will be able to win a vote under *any* majority voting rule, as long as it is an opting-out rule. If the principal of an inferior party has the control right she will be able to prevent the opposed party from winning as long as the own initial numerical strength is equal or higher than that of the other party.

Let us now come to other mechanisms of interaction which will lead us one step beyond control rights over the timing of the vote towards control rights over interaction mechanisms.

3.5.2 Unconcentrated Interaction

In the previous section the mechanism of interaction allowed for the concentration of individual efforts with the important result that numbers were

more important than effectiveness. Political campaigning is however often conducted in such a way that an expression of opinion is a *public good* (or a public bad, depending on which side one is). There then are spillover-effects from efforts to convince somebody of the other party.

Let us now have a look at support attrition here.⁹ We again assume away moral superiority. The linear homogenous system of differential equations

$$\text{M2: } \begin{cases} \dot{x} = -gxy, \\ \dot{y} = -hxy \end{cases} \quad (24)$$

describes the pure attrition of support. g and h are the success probabilities which are derived in the same way as c and d . Here we get

$$\begin{aligned} gxy\dot{y} - hyx\dot{x} &= 0 \\ \Leftrightarrow \frac{dx}{dy} &= \frac{g}{h} \end{aligned} \quad (25)$$

from which by separating the variables and integrating from (x_0, y_0) to $(x(t), y(t))$ we obtain the state equations for the initial-value problem $x(0) \equiv x_0$ and $y(0) \equiv y_0$

$$g \int_{y_0}^{y(t)} xy dy = h \int_{x_0}^{x(t)} yx dx \quad (26)$$

from which follows

$$g(y(t) - y_0) = h(x(t) - x_0). \quad (27)$$

The system can be written as

$$y(t) = y_0 + \frac{h}{g}(x(t) - x_0) \quad (28)$$

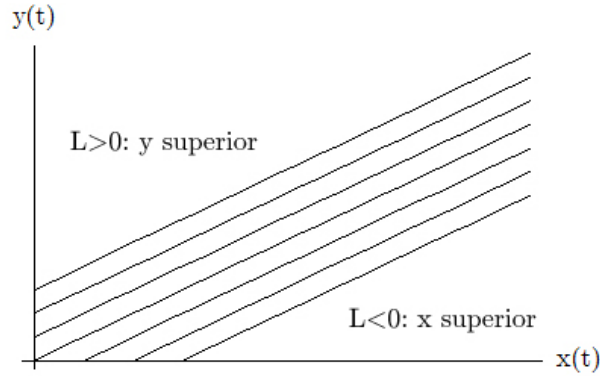
and

$$x(t) = x_0 + \frac{g}{h}(y(t) - y_0). \quad (29)$$

The trajectories are straight parallel lines, as depicted in Figure 3, the one going through the coordinate origin being that leading to a stalemate.

⁹ This part is based on Coleman (1982), 115–117.

Fig. 3: Support Attrition Trajectories (28), (29)

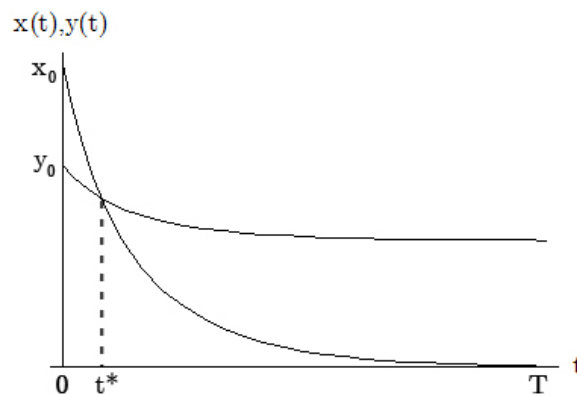


We now define $gy_0 - hx_0 \equiv L$. For $L = 0$ we obtain

$$\frac{y_0}{x_0} = \frac{h}{g} \quad (30)$$

which means that in order to stalemate an opposing party with n times as many supporters one needs to be only n times as effective in eroding support: y is superior if $(y_0/x_0) > (h/g)$, while x is said to be superior if $(y_0/x_0) < (h/g)$. This is the superiority condition as formulated in Definition 1.

The graph of the general solution is shown in Figure 4.¹⁰

Fig. 4: General Solutions of (24) for some $L > 0$ 

¹⁰ The values in this example are $x_0 = 30$, $y_0 = 20$, $g = 0.2$ and $h = 0.1$.

The support attrition rate for one party is here proportional to the number of opposed supporters as well as to the own. This leads to another result which, interestingly, contradicts intuition.

Proposition 3 (Incomplete): *Confidential talks and plenary debates are equivalent interaction mechanisms.*

Proof (Incomplete): In confidential talks the for each support loss of faction x is ■

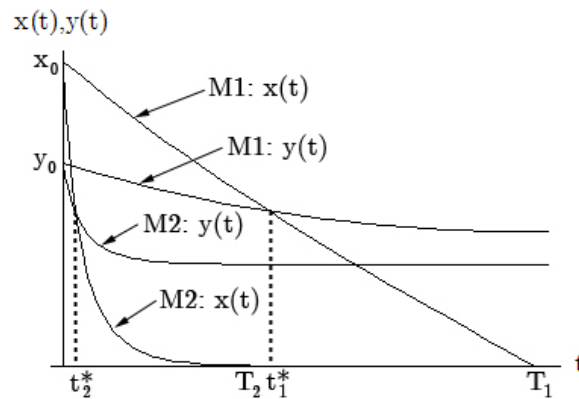
4 Procedures and Control Structures (Incomplete)

Control structures are an important aspect of political competition. Considering interaction mechanisms and the dimension of time several new possibilities appear to influence the outcome of a vote, namely (exclusive) control of the timing of the vote and of who is allowed to address whom in what way.

4.1 Superiority Independent of the Mechanism

Let us now compare both models in Figure 5 and illustrate the effects of different control structures.¹¹

Fig. 5: Comparison of general solutions of (15) and (24) for some $K > 0$ and $L > 0$



¹¹ The values used here are $x_0^{M1} = x_0^{M2} = 30$, $y_0^{M1} = y_0^{M2} = 20$, $b = g = 0.4$ and $c = h = 0.1$.

If the interaction mechanism were M1 x would win a SMV from $t = 0$ until t_1^* . Exactly in t_1^* there would be a draw, but at each point in time afterwards y would win a SMV. From T_1 on y would moreover win a vote under the OUV rule. From Proposition 2 follows that if principal x would have the control rights over the timing of, say, a SMV vote it would not make any sense to vote later than t_1^* while principal y would prefer to vote later than this. If the interaction mechanism were M2 y would win an SMV vote any time from t_2^* on and achieve an OUV unanimity at each point in time later than T_2 . Let us have a look at a fictitious example.

Example 1: Imagine a committee with 50 members were divided into two factions and the numerical values to be those used for Figure 5. The committee procedures foresee a simple majority vote to be taken on the principals' motions. We assume principal y to be the chairwoman of the committee and in possession of the control rights of the timing of the vote and the interaction mechanism. Let us further assume information to be asymmetric in the sense that principal y does *not* know about the differences between both mechanisms (she wrongfully believes M2 to be equivalent to M1) while principal x knows the attrition rates are different. Faction y is superior because of a moral advantage compensating the numerical advantage and peer pressure of faction x . In $t = 0$ the debate starts with exchanges of opinion in the traditional way—plenary debate—the support attrition curves thus following M2. Principal y expects to win a vote at any $t \in]t_2^*, \infty[$ and uses her control right to fix the timing of the SMV at T_2 . Upon this announcement principal x calls for a break of the session, which we assume is granted by the chair. During the break the agents walk around in the lobby. The agents of party x use the opportunity to address every remaining active agent of party y with their quite convincing arguments, working in groups and concentrating their efforts. Whenever a member of y signals to be convinced the agents having convinced her join other groups thus raising their chances of success. Unknown to the chairwoman the support attrition curves perform according to M1. On the end of the break at T_2 the chairwoman decides to take the vote, but contrary to her expectations the x -party wins the vote with a surprising majority of $x(T_2) - y(T_2) > 0$ votes. The scandal is perfect.

4.2 Superiority depending from the Mechanism

In the case forming the basis for the comparison in Figure 5 and the above example the allocation of control rights over the interaction mechanism does not influence whether a party is superior or not, because y is superior under both mechanisms. There are however cases in which one party would be superior under one mechanism but inferior under the other. The allocation of control rights then determines not only t^* and T but also who will get the upper hand in campaigning. This observation leads to Proposition 3.

Proposition 3: *The control structure ceteris paribus determines the status of superiority if $\sqrt{c/b} < y_0/x_0 < h/g$ or $h/g < y_0/x_0 < \sqrt{c/b}$.*

Proof: Superiority changes if under M1 $K > 0$ (y superior) while under M2 $L < 0$ (x superior) (Case I) and vice versa (Case II). The conditions for a change of superiority that need to be fulfilled are

$$\text{Case I: } \sqrt{\frac{c}{b}} < \frac{y_0}{x_0} \quad \wedge \quad \frac{h}{g} > \frac{y_0}{x_0} \quad (31)$$

or

$$\text{Case II: } \sqrt{\frac{c}{b}} > \frac{y_0}{x_0} \quad \wedge \quad \frac{h}{g} < \frac{y_0}{x_0} \quad (32)$$

for a given combination of positive initial values x_0, y_0 and of positive effectiveness coefficients b, c, g, h . As long as these inequalities are fulfilled an inferior party can become superior if it can choose the mechanism favouring its particular combination of initial numeric strength and effectiveness. This is only possible if there are control rights over the mechanisms and if it has the control rights over choosing the interaction mechanism.¹² ■

5 Conclusion (Incomplete)

Of course, the facets of real-life political interaction are quite complex and depending from factors that are hard to assess before the fact. Emotions, group dynamics, hidden agendas, log-rolling, strategic disinformation or even bribes may all influence political agents and thus the outcome of a vote. While the model developed here is simple and suggestive, rather than refined

¹² It does not seem to be too strong an assumption to rule out the case that the opposed principal erroneously chooses the wrong mechanism.

and definitive, it serves as a paradigmatic wedge by which the importance of interaction, and control over it, is exposed and is easier made the vehicle for further analysis. It surely generates a reasonable set of qualitative behaviors with predictive value.

It makes sense to interpret interaction mechanisms as microtechnologies of political competition which differ in the ability to concentrate one's efforts on the opposed active agents. While this is possible in some mechanisms it is ruled out in the others. If it is rational for supporters to be ignorant they might be motivated by external incentives provided by political entrepreneurs. If incentive structures are rigid political competition will be shifted to the field of control rights, i.e. to procedural questions and the exploitation of rules of order for one's benefit. Needless to say there is a good deal of opportunism involved.

Modern democratic procedures do in most cases not contain exclusive control rights prohibiting for instance confidential two-person talks. It appears hardly democratic if one principal could dictate the timing of the vote and the interaction mechanism. But that does not mean control can not be exercised. There is good reason to assume not all political entrepreneurs have a thorough understanding of how democratic procedures can be exploited with aim to influence the outcome of a vote, like in the case of the agenda, and therefore fail to understand how control slips through their fingers. What we are trying to explain are much subtler strategies.

Against a relatively badly informed principal control over the timing of the vote can for instance be achieved by just putting topics at the bottom of the agenda in order to exploit an unwillingness to debate for long because of fatigue. Control over the interaction mechanism can be acquired by postponing a debate, the strategic use of adjournments or even in short breaks. Well-informed opponents will counter such moves which often results in squabbling over the proper procedures.

The use of adjournments appears as such a strategy. If a political entrepreneur leading a large mediocre motivated faction is confronted with a small network consisting highly effective agents it would be dangerous to expose the own agents either to public statements, where one good orator can potentially influence every member of an assembly, or to confidential talks. The efficient strategy is then to round up selected supporters of the other side with several own agents in order to raise their chance of success. Remember that efforts are aimed at destroying the opposed network.

The timing of the vote can be up to a certain extent controlled by putting a topic at the top or the bottom of the order of the day. In the first case

the willingness of the agents to discuss will most probably be greater and the debate can be more easily be stretched. The second case exploits the unwillingness to debate for long after a arduous session. Exploiting such human weaknesses is exactly what makes up the dark side of politics.

Last but not least there are strategies how control over the interaction mechanism can be exercised. It seems that the way conferences and meetings are organized confers control possibilities and this explains why for instance party leaderships organize conferences for days on end to decide questions of principle. Such conferences open many possibilities to change the interaction mechanisms, alternating plenary debates, meetings in committees and confidential talks, and thus many possibilities to influence the outcome of an interaction process.

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Appendix

A.1

In order to find the general solution we solve the system (15). We start by writing it in matrix form:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -b \\ -c & 0 \end{pmatrix}}_{=:A} \begin{pmatrix} x \\ y \end{pmatrix}. \quad (\text{A-1})$$

The Eigenvalues of A are $\lambda_1 = \sqrt{bc}$ and $\lambda_2 = -\sqrt{bc}$ with corresponding Eigenvectors

$$z_1 = \begin{pmatrix} \sqrt{b} \\ -\sqrt{c} \end{pmatrix} \quad \text{and} \quad z_2 = \begin{pmatrix} \sqrt{b} \\ \sqrt{c} \end{pmatrix} \quad (\text{A-2})$$

which indicate a saddle point. A fundamental system is

$$\begin{pmatrix} \sqrt{b}e^{\sqrt{b}ct} \\ -\sqrt{c}e^{\sqrt{b}ct} \end{pmatrix}, \begin{pmatrix} \sqrt{b}e^{-\sqrt{b}ct} \\ \sqrt{c}e^{-\sqrt{b}ct} \end{pmatrix}. \quad (\text{A-3})$$

To find the solution for the initial-value problem we solve

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \lambda \begin{pmatrix} \sqrt{b} \\ -\sqrt{c} \end{pmatrix} e^{\sqrt{b}c0} + \mu \begin{pmatrix} \sqrt{b} \\ \sqrt{c} \end{pmatrix} e^{-\sqrt{b}c0} \quad (\text{A-4})$$

and obtain

$$\lambda = \frac{x_0}{2\sqrt{b}} - \frac{y_0}{2\sqrt{c}} \quad \text{and} \quad \mu = \frac{x_0}{2\sqrt{b}} + \frac{y_0}{2\sqrt{c}}. \quad (\text{A-5})$$

Here we acquire the solutions

$$x(t) = x_0 \frac{1}{2} (e^{\sqrt{b}ct} + e^{-\sqrt{b}ct}) - y_0 \frac{\sqrt{b}}{\sqrt{c}} \frac{1}{2} (e^{\sqrt{b}ct} - e^{-\sqrt{b}ct}) \quad (\text{A-6})$$

$$= x_0 \cosh(\sqrt{b}ct) - y_0 \frac{\sqrt{b}}{\sqrt{c}} \sinh(\sqrt{b}ct) \quad (\text{A-7})$$

and

$$y(t) = y_0 \frac{1}{2} (e^{\sqrt{b}ct} + e^{-\sqrt{b}ct}) - x_0 \frac{\sqrt{c}}{\sqrt{b}} \frac{1}{2} (e^{\sqrt{b}ct} - e^{-\sqrt{b}ct}) \quad (\text{A-8})$$

$$= y_0 \cosh(\sqrt{b}ct) - x_0 \frac{\sqrt{c}}{\sqrt{b}} \sinh(\sqrt{b}ct). \quad (\text{A-9})$$

A.2

(See e.g. Schikowski 1971, 151-153). The system (24) can be written in matrix form:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -g \\ -h \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} y \\ x \end{pmatrix} \quad (\text{A-10})$$

By substituting X for gy and Y for hx we get the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \underbrace{\begin{pmatrix} X \\ Y \end{pmatrix}}_{=:B} \begin{pmatrix} x \\ y \end{pmatrix} \quad (\text{A-11})$$

which can now be solved. Let us define $\dot{X} \equiv g\dot{y}$ and $\dot{Y} \equiv h\dot{x}$, so that obviously

$$\dot{X} = -XY = \dot{Y}. \quad (\text{A-12})$$

From

$$\int_{y_0}^{y(t)} dY = \int_{x_0}^{x(t)} dX \quad (\text{A-13})$$

follows directly

$$Y = X + v \quad (\text{A-14})$$

where $v \equiv y_0 - x_0$. Inserting (A-14) into (A-12) leads to

$$-\frac{dX}{X^2 - vX} = dt. \quad (\text{A-15})$$

We can now integrate both sides. Substituting u for t we get

$$\int_{X(0)}^{X(t)} -\frac{dX(u)}{X^2(u) - cX(u)} = \int_0^t du. \quad (\text{A-16})$$

Recall that

$$\int \frac{dx}{ax^2 + bx + c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} & \text{for } 4ac - b^2 > 0, \\ \frac{2}{\sqrt{b^2-4ac}} \operatorname{artanh} \frac{2ax+b}{\sqrt{b^2-4ac}} & \text{for } 4ac - b^2 < 0, \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| & \text{for } 4ac - b^2 < 0. \end{cases}$$

Accordingly, we get for (A-16)

$$\frac{1}{v} \ln \frac{X(t)}{X(t)+v} - w = t \quad (\text{A-17})$$

with

$$w \equiv \frac{1}{v} \ln \frac{X(0)}{X(0) + v}. \quad (\text{A-18})$$

We acquire

$$X(t) = \frac{v e^{v(t+w)}}{1 - e^{v(t+w)}} \quad (\text{A-19})$$

and with (A-14)

$$Y(t) = \frac{1}{1 - e^{v(t+w)}}. \quad (\text{A-20})$$