

Redistributive Politics with Distortionary taxation

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Very Preliminary
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Abstract

We extend the discussion of redistributive politics across electoral systems to allow for taxation to be distortionary. We build on the model put forward by Lizzeri and Persico (2001 and 2005). We allow politicians to choose between three options: 1) full taxation and redistribution; 2) partial taxation and redistribution; and 3) zero taxes and no redistribution. We show that increasing the candidates' flexibility in making promises to the electorate may be welfare enhancing when the degree of convexity of the distortionary cost of taxation rises above a certain threshold. We also find that the equilibrium under first-past-the-post is independent of distortions. Thus proportional representation leads to higher (resp. lower) expected welfare than plurality rule when the degree of convexity of tax distortions is high (resp. low).

Keywords: Redistributive Politics, Distortionary Taxation, Electoral Rules

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1 Introduction

The determinants of the size and composition of public spending have been the focus of analysis of many recent papers in political economy. In particular, papers by Lizzeri and Persico (2001 and 2005) highlight the importance of the extent to which the costs and benefits of spending can be targeted to subgroups of the population. They analyze the inefficiency that arises when redistributive policies targeted to particular subsets of the population are overprovided at the expense of spending that can be targeted to subgroups of the population. Targetability is valuable to politicians whose objective is to get elected but it has no social value. The trade-off Lizzeri and Persico focus on is that between a public project that benefits the whole population and pure, targeted, redistribution.

We reinterpret their model in the context of redistributive politics with distortionary taxation. Given the inefficiency of taxation, no taxation can be reinterpreted as the efficient policy (it is similar to providing a public good; a policy that benefits everybody but that is not targetable) while taxation is inefficient but enables politicians to have a budget from which they can make targeted promises. For the effect of distortionary taxation to be analyzable, we need to give politicians the choice between at least three different tax rates. Hence, we extend the model of Lizzeri and Persico to give candidates the choice between pure redistribution, partial redistribution and zero taxes.

We first solve the model under proportional representation and then turn to the case of majoritarian, first-past-the-post politics.

Results

We first show that extend Lizzeri and Persico (2005)'s result about the linearity of equilibrium payoffs in these types of redistributive games carries through to the case of three options.

We then solve for the equilibrium of the game under PR. Our results show that previous results in this literature underestimate the costs of inefficient policies: if the distortionary cost of taxes rises above (resp. remains below) a certain threshold, giving politicians more flexibility in their policy choices is welfare enhancing (resp. welfare reducing) for voters. We also find that equilibrium probability of selecting any of the three choices is a function of the tax distortions under PR.

Next, we solve for the equilibrium under plurality rule and find that the probabilities of selecting any of the three options are independent of the distortions under FPTP. This implies the following comparative politics prediction: PR yields a higher (resp. lower) level of welfare than FPTP when

distortions are high (resp. low).

The rest of the paper is structured as follows. Section 2 presents the basic model. Section 3 solves for the equilibrium under PR and compares the economy's welfare under the present setup to that under the two-option scenario of Lizzeri and Persico (2001). Section 4 solves for the equilibrium under FPTP and compares this equilibrium with that under PR. The last section concludes. The Appendix contains some of the proofs.

2 The Basic Model

2.1 Economy and players

There are two candidates, A and B . The electorate is made of a continuum E of total mass 1. Each voter is endowed with one unit of money. Politicians can tax voters' endowment and then make promises. These promises are subject to an economy-wide budget constraint that constrains them to make balanced-budget policy pledges. These promises are binding.

Candidates choose their offer from any of the following three policies. 1) no taxes (the efficient policy), 2) a small tax rate (each citizen is taxed a share $1 - \alpha$ of his endowment, 3) a high tax rate (each citizen is taxed his full endowment¹). At the same time they announce the tax-rate, politicians make individual binding promises to voters under the constraint that the sum of transfers must equal the total of money collected net of the cost of collection. Thus promises are credible. We assume distortionary taxation: there is a cost to collect taxes: only part of the income taxed is available for redistribution. In the case all income is taxed only a budget of λ is available for redistribution. Similarly, if a share $1 - \alpha$ is taxed we assume that the budget available for redistribution is $(1 - \alpha) \lambda_\alpha$. We assume that $1/2 < \lambda \leq \lambda_\alpha \leq 1$. This ensures that in equilibrium all three policy choices may be used and also means that the cost of collecting taxes increases with the tax rate.

The timing of the game is as follows:

1. Each candidate makes binding and credible promises to voters, simultaneously and independently;
2. After observing the two candidates' offers, voters cast their ballot;

¹This is a renormalization. The high tax rate could correspond to only a percentage of voters' endowment leaving the rest to voters.

3. Vote shares determine the electoral outcome and payoffs are realized.

We use a reduced-form mapping from the legislature to the executive: the probability that the policy chosen by a candidate is the implemented one is an increasing function of the vote share of that candidate. This implies in turn that each voter votes sincerely, that is, casts his ballot in favor of the candidate that promises him the greatest utility.

We solve the game under proportional representation (PR), and plurality rule of first-past-the-post (FPTP). Under PR, the mapping from the legislature to the executive (and the spoils from office) is directly proportional to vote shares. Thus candidates maximize the share of the total votes.

Under FPTP, the mapping from the legislature to the executive is such that a candidate's payoff is 1 if he/she wins a majority of the votes and zero otherwise. In case of a tie each candidate's payoff is 1/2. Hence, under plurality rule, Candidates maximize their probability of winning a majority of the votes.

2.2 Game and politicians' strategies

A pure strategy for a candidate specifies the tax rate he chooses. In the event he chooses a positive tax rate, a pure strategy specifies a promise of a transfer to each voter. Formally a pure strategy is a function $\Sigma : E \rightarrow [0, +\infty)$, where $\Sigma(e)$ represents the consumption promised to voter e . The function e must satisfy one of three conditions (depending on the tax rate chosen by the candidate): either $\Sigma(e) = 1$ for all e 's if the candidate chooses a zero tax rate, or $\int_e \Sigma(e) de = (1-\alpha)\lambda_\alpha$, which is the balanced budget condition when the intermediate tax rate is chosen (in that case, $\Sigma(e) - (1-\alpha)$ represents the utility promised to voter e , or $\int_e \Sigma(e) de = \lambda$, which is the balanced budget condition when a candidate taxes all income (in that case $\Sigma(e) - 1$ represents the utility promised to voter e).

For values of λ and λ_α satisfying $1/2 \leq \lambda \leq \lambda_\alpha \leq 1$, there is no equilibrium in pure strategies². At equilibrium, both candidates will be using mixed strategies. We discuss the case of equilibria in simple strategies of the following form. Candidate i chooses the tax rates with probabilities p , p_α and $1 - p - p_\alpha$ respectively for no tax, intermediate taxes and full taxation. When candidates redistribute, they draw promises to all voters from the same distributions F_R^i and F_α^i . due to the high number of voters, F_R^i and F_α^i are the empirical distributions of transfers by candidate i to

²The argument is standard. See Lizzeri and Persico (2001) for details.

voters.

2.3 Fundamental property: Linearity of Equilibrium payoffs

This section is an extension of section 3 in Lizzeri and Persico (2005). They were the first to show that equilibrium payoffs in this type of game of redistribution are piece-wise linear.

Consider an equilibrium with ex-ante equal treatment and let the functions $W_R^*(x)$ and $W_\alpha^*(x)$ denote the equilibrium probability of winning a vote with a promise of utility of x to a voter when you respectively tax all or a share α of the income.

There is a simple intuition behind the linearity of the equilibrium payoffs, $W_R^*(x)$ and $W_\alpha^*(x)$. Let us focus first on $W_R^*(x)$. $W_R^*(x)$ represents the expected benefit of making a promise of a transfer of x to a voter. In equilibrium, the expected benefit of this promise must equal its cost. The cost of spending x is the shadow cost associated with the budget constraint, which represents the opportunity cost of funds for a candidate. This opportunity cost is linear in x . Therefore, $W_R^*(x)$ must also be linear in x .

We prove below that the support of W_R^* is given by $[0, 2\lambda]$. Then each candidate solves the following maximization problem:

$$\text{Max}_F \int_0^{2\lambda} W_R^*(x) dF(x) \text{ s.t. } \int_0^{2\lambda} x dF(x) = \lambda.$$

The associated Langrangian is:

$$\mathcal{L} = \int_0^{2\lambda} (W_R^*(x) + \gamma(\lambda - x)) dF(x).$$

Let γ^* be the optimal of the Lagrange multiplier. Since F_R^* maximizes the Lagrangian, it must be that $W_R^*(x) - \gamma^*x$ is maximal and constant on the support of F_R^* . We have $W_R^*(0) - \gamma^* \cdot 0 = 0$. This leads to $0 = W_R^*(2\lambda) - \gamma^* \cdot 2\lambda = 1 - \gamma^* \cdot 2\lambda$. Hence $\gamma^* = \frac{1}{2\lambda}$ and $W_R^*(x) = \frac{x}{2\lambda}$ for all x on the support of F_R^* .

Turning to the support of W_R^* , suppose the minimum were some constant $k > 0$. Given the structure of the game, offering k yields a payoff of zero: k is associated with a vote share of 0. Then this cannot be an equilibrium: we must have that the minimum of W_R^* is 0.

To see why 2λ is the maximum of the support of W_R^* , notice that because W_R^* is linear in x and $W_R^*(0) = 0$, we must have that $W_R^*(x) = \kappa x$, $\kappa > 0$, for any x belonging to its support. Label the maximum of the support as M . M must be such that $W_R^*(M) = \kappa M = 1$. Thus $M = 1/\kappa$. We also know that the budget constraint implies that $\int_0^{1/\kappa} x dF(x) = \lambda$ and that in equilibrium this

strategy must yield a candidate a share $1/2$ of the votes. Thus:

$$1/2 = \int_0^{1/\kappa} W_R^*(x) dF(x) = \int_0^{1/\kappa} \kappa x dF(x) = \kappa \int_0^{1/\kappa} x dF(x) = \kappa \lambda$$

which finally implies that $M = 2\lambda$.

Following exactly the same line of reasoning implies that W_α^* is also linear (on its support) and that the support is $[\alpha, \alpha + (1 - \alpha)\lambda_\alpha]$.³

3 Equilibrium under Proportional Representation

Under PR, each candidate maximizes his share of the votes. In the Appendix, we prove that the equilibrium of the game under PR is given by:

Proposition 1 *The Nash equilibrium under proportional representation is such that full redistribution involves offers between $[0, K_1]$ and $[K_3, 2\lambda]$, partial redistribution involves offers between $[\alpha, K_2]$ and $[1, K_3]$ with:*

$$\begin{cases} K_1 = 2\lambda - [\alpha + 2\lambda_\alpha(1 - \alpha)] < \alpha \\ K_3 = \alpha + 2\lambda_\alpha(1 - \alpha) > 1 \\ K_2 = 2\alpha - 1 + 2(1 - \alpha)\lambda_\alpha \in (\alpha, 1) \end{cases}$$

The probability that candidates choose the no or partial taxation option are given respectively by:

$$\begin{cases} p = \frac{\alpha - \lambda(1 - \lambda_\alpha) + \lambda_\alpha(2 - 3\alpha) - 2(1 - \alpha)(\lambda_\alpha)^2}{\lambda} \\ p_\alpha = \frac{\alpha - \lambda + 2(1 - \alpha)\lambda_\alpha}{\lambda} - p \end{cases}$$

Graphically, the game can be represented as follows:

³Both the minimum and the maximum of the support include α because $W_\alpha^*(x)$ denotes the equilibrium probability of winning a vote with a promise of *utility* of x when a share α of the income is taxed.

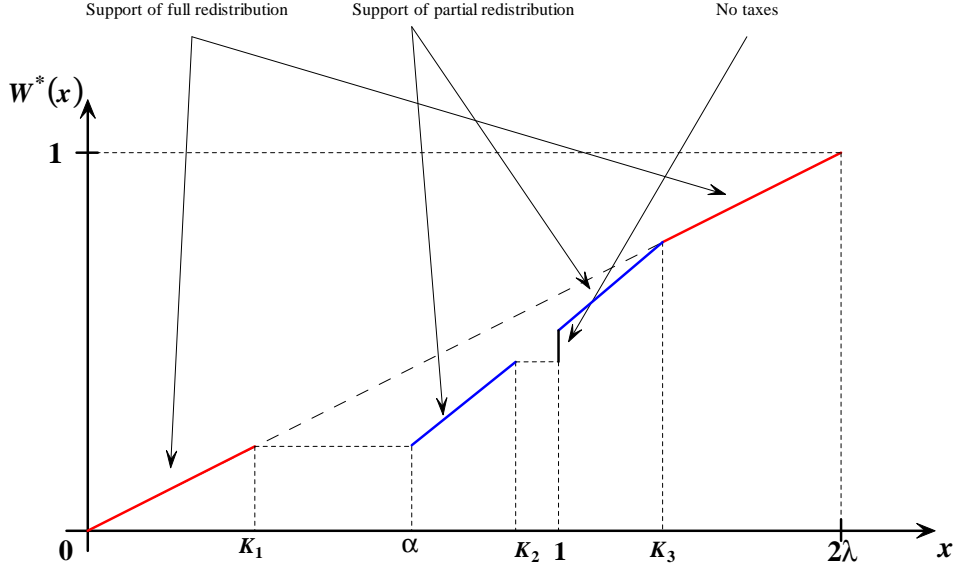


Figure 1: Equilibrium Winning Probabilities

3.1 Comparative Statics

By direct differentiation, we have:

Proposition 2

$$\left\{ \begin{array}{l} \frac{\partial K_1}{\partial \lambda} = 2 \\ \frac{\partial \lambda}{\partial K_3} = \frac{\partial K_2}{\partial \lambda} = 0 \\ \frac{\partial p}{\partial \lambda} = -\frac{\alpha + 2\lambda_\alpha - 3\alpha\lambda_\alpha - 2\lambda_\alpha^2(1-\alpha)}{\lambda^2\lambda_\alpha} \leq 0 \\ \frac{\partial p_\alpha}{\partial \lambda} = \frac{\alpha + 2\lambda_\alpha - 4\alpha\lambda_\alpha - 4\lambda_\alpha^2(1-\alpha)}{\lambda^2\lambda_\alpha} \leq 0 \end{array} \right. \text{ and } \left\{ \begin{array}{l} \frac{\partial K_1}{\partial \lambda_\alpha} = -\frac{\partial K_3}{\partial \lambda_\alpha} = -2(1+2\alpha) < 0 \\ \frac{\partial K_2}{\partial \lambda_\alpha} = 2(1-\alpha) > 0 \\ \frac{\partial p}{\partial \lambda_\alpha} = \frac{-2\lambda_\alpha^2(1-\alpha) - \alpha + \lambda}{\lambda\lambda_\alpha^2} \leq 0 \\ \frac{\partial p_\alpha}{\partial \lambda_\alpha} = \frac{-4\lambda_\alpha^2(1-\alpha) - \alpha + \lambda}{\lambda\lambda_\alpha^2} \leq 0 \end{array} \right.$$

Proof. See Appendix. ■

3.2 Welfare

We compare the expected welfare of voters under our setup with that under the setup of Lizzeri and Persico (2001). In order to do this we need to rewrite the equilibrium in that paper using our formulation. It is easy to check that this yields:

$$\begin{aligned} K_1 &= 2\lambda - 1 \\ K_3 &= 1 \\ \tilde{p} &= \frac{1-\lambda}{\lambda} \end{aligned}$$

Thus, expected utility under the $LP01$ scenario is:

$$EU(LP01) = \underbrace{\tilde{p}\lambda}_{\text{prob of redistributing} \cdot \text{budget}} + \underbrace{1-\tilde{p}}_{\text{prob of laissez-faire} \cdot 1} = \frac{3\lambda - \lambda^2 - 1}{\lambda}$$

Whereas under our setup expected utility $EU(CS)$ is given by:

$$\underbrace{p\lambda}_{\text{prob of full red.} \cdot \text{budget}} + \underbrace{1-p-p_\alpha}_{\text{prob of laissez-faire} \cdot 1} + \underbrace{p_\alpha[(1-\alpha)\lambda_\alpha + \alpha]}_{\text{prob of partial red.} \cdot \text{value of partial red.}}$$

In the appendix, we prove:

Proposition 3 *When the convexity of the distortionary cost of taxation exceeds a certain threshold, giving politicians more flexibility (in terms of more policy choices from which to choose) increases the voters' welfare.*

Proof. See Appendix. ■

This proposition highlights that previous contributions have underestimated the distortionary costs of inefficient policies.

4 Equilibrium under Plurality Rule (to be completed)

Under British style FPTP, all each candidate care about is winning, no matter how thin is his margin of victory. Thus, the payoffs are 1 in case of victory, 0 in case of defeat and 1/2 if there is a tie. In equilibrium we must have a tie. Thus equilibrium payoffs are given by (1/2, 1/2). We then have:

Proposition 4 *Bounds K_1 , K_2 and K_3 are the same as under PR. The probabilities of selecting*

$$\text{any of the three possible policies are: } \begin{cases} 1-p-p_\alpha = 1/2; \\ p = p_\alpha = 1/4. \end{cases}$$

Thus, the equilibrium is independent of the level and the degree of convexity of distortions

Proof. See Appendix. ■

5 Comparative Politics (to be completed)

Comparing propositions 1 and 3 above, we have:

Proposition 5 *PR associated to higher social welfare than FPTP when distortions are more important.*

6 Conclusion

This paper has extended the previous analyses of redistributive politics across electoral systems to allow for taxation to be distortionary.

The results show that PR may be preferred to FPTP by voters when taxes are distortionary because the incentives politicians face under PR push them to take the cost of distortionary taxation into account.

It would be interesting to test empirically these predictions. We plan to carry out such test in future research. We also plan to extend the model to one with a continuum of policy choices. This should allow us to pinpoint more precisely the mapping between distortionary taxation and electoral promises and obtain further testable theoretical predictions.

7 References

References

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8 Appendix: Proofs

8.1 Proof of proposition 1

To prove proposition 1, we compute the equilibrium conditions that characterize the equilibrium under PR.

8.1.1 Full targeted redistribution

Remember that F_R^* is the equilibrium cdf associated with full targeted redistribution. This cdf has support on $[0, K_1] \cup [K_3, 2\lambda]$, for some $0 < K_1 < K_3 < 2\lambda$. Thus, by the linearity property of $W^*(x)$, we must have that:

$$\begin{aligned} W^*(x) &\equiv \frac{x}{2\lambda} = (1 - p - p_\alpha) F_R^*(x) \\ &\Leftrightarrow \\ F_R^*(x) &= \frac{x}{2\lambda(1 - p - p_\alpha)} \end{aligned}$$

for all $x \leq K_1$, and:

$$\begin{aligned} W^*(x) &\equiv \frac{x}{2\lambda} = (1 - p - p_\alpha) F_R^*(x) + p + p_\alpha \\ &\Leftrightarrow \\ F_R^*(x) &= \frac{x/2\lambda - (p + p_\alpha)}{(1 - p - p_\alpha)} \end{aligned}$$

for all $x \geq K_3$.

To pin down the values of K_1 and K_3 , we make use of the continuity of F_R^* and of the budget constraint. By the continuity of F_R^* , we must have that $F_R^*(K_1) = F_R^*(K_3)$, thus our first equilibrium condition yields:

$$K_1 = K_3 - 2\lambda(p + p_\alpha) \tag{1}$$

The second equilibrium condition is derived from the balanced-budget constraint:

$$\int_0^{K_1} x dF_R^*(x) + \int_{K_3}^{2\lambda} x dF_R^*(x) = \lambda$$

which yields:

$$(K_1)^2 - (K_3)^2 = -4\lambda^2(p + p_\alpha) \tag{2}$$

Together, conditions (1) and (2) yield:

$$\begin{cases} K_1 = \lambda(1 - p - p_\alpha) \\ K_3 = 2\lambda - K_1 = \lambda(1 + p + p_\alpha) \end{cases} \tag{3}$$

8.1.2 Partial Redistribution

There are also two conditions associated to the candidates' offer of partial redistribution with funds worth $(1 - \alpha) \lambda_\alpha$.

Remember that F_α^* is the cdf in terms of *utils* each voter gets from partial redistribution. The support of F_α^* is $[\alpha, K_2] \cup [1, K_3]$. To simplify notation, label the slope of W_α^* as s . The equilibrium probability of winning a vote with an offer in $[\alpha, K_2]$ and $[1, K_3]$, respectively, is:

$$\begin{aligned} W^*(x) &\equiv s(x - \alpha) = (1 - p - p_\alpha) F_R^*(K_1) + p_\alpha F_\alpha^*(x) \\ &\Leftrightarrow \\ F_\alpha^*(x) &= \frac{s(x - \alpha) - (1 - p - p_\alpha) F_R^*(K_1)}{p_\alpha} \end{aligned}$$

for all $x \in [\alpha, K_2]$, and:

$$\begin{aligned} W^*(x) &\equiv s(x - \alpha) = (1 - p - p_\alpha) F_R^*(K_1) + p + p_\alpha F_\alpha^*(x) \\ &\Leftrightarrow \\ F_\alpha^*(x) &= \frac{s(x - \alpha) - (1 - p - p_\alpha) F_R^*(K_1) - p}{p_\alpha} \end{aligned}$$

for all $x \in [1, K_3]$.

Exploiting the continuity of F_α^* , which implies

$$F_\alpha^*(K_2) = F_\alpha^*(1),$$

yields our third equilibrium condition:

$$K_2 - \alpha = 1 - \alpha - \frac{p}{s} = 1 - \alpha - \frac{p}{p + p_\alpha} (K_3 - \alpha) \quad (4)$$

The fourth condition that pinpoints the equilibrium of this game is again derived through the budget constraint associated to partial redistribution:

$$\int_0^{K_2 - \alpha} x dF_\alpha^*(x) + \int_{1 - \alpha}^{K_3 - \alpha} x dF_\alpha^*(x) = (1 - \alpha) \lambda_\alpha$$

That is:

$$(K_2 - \alpha)^2 + (K_3 - \alpha)^2 - (1 - \alpha)^2 = \frac{2p_\alpha (1 - \alpha) \lambda_\alpha}{s} \quad (5)$$

The fifth and last condition that we need to fully characterize the equilibrium is derived from the vote shares at equilibrium, which must be equal to each other and each equal to 1/2. These

vote shares are given by:

$$\begin{aligned}
S_M &= 1/2(1 - p - p_\alpha) + p(1 - F_R^*(K_3)) + p_\alpha(1 - F_R^*(K_3)) \\
S_G &= (1 - p - p_\alpha) F_R^*(K_3) + 1/2p + p_\alpha F_\alpha^*(K_2) \\
S_{\alpha G} &= (1 - p - p_\alpha) F_R^*(K_1) + p(1 - F_\alpha^*(K_2)) + 1/2p_\alpha
\end{aligned}$$

The fifth and last condition is thus:⁴

$$\begin{aligned}
F_R^*(K_3) &= 1/2 \\
F_\alpha^*(K_2) &= 1/2
\end{aligned}$$

This implies in turn that the distance between K_2 and α is equal to that between K_3 and 1. We thus have:

$$K_2 - \alpha = K_3 - 1 \quad (6)$$

The system of five equations in five unknowns ($K_1, K_2, K_3, P_G, P_{\alpha G}$) we need to solve is:

$$\left\{ \begin{array}{l}
K_1 = \lambda(1 - p - p_\alpha) \\
K_3 = \lambda(1 + p + p_\alpha) \\
K_2 - \alpha = 1 - \alpha - \frac{p}{p + p_\alpha} (K_3 - \alpha) \\
(K_2 - \alpha)^2 + (K_3 - \alpha)^2 - (1 - \alpha)^2 = \frac{2p_\alpha}{p + p_\alpha} (K_3 - \alpha) (1 - \alpha) \lambda_\alpha \\
K_2 - \alpha = K_3 - 1
\end{array} \right.$$

The unique solution to this system is the one of proposition 1.

8.2 Proof of proposition 2

By simple differentiation and some algebraic manipulations, we have:

- $sgn\left(\frac{\delta p}{\delta \lambda}\right) = -sgn\left(\alpha + 2\lambda_\alpha - 3\alpha\lambda_\alpha - 2\lambda_\alpha^2(1 - \alpha)\right)$. But:

$$\begin{aligned}
&\alpha + 2\lambda_\alpha - 3\alpha\lambda_\alpha - 2\lambda_\alpha^2(1 - \alpha) \\
&= \alpha(1 - \lambda_\alpha) + 2\lambda_\alpha(1 - \alpha) - 2\lambda_\alpha^2(1 - \alpha) \\
&= \alpha(1 - \lambda_\alpha) + 2\lambda_\alpha(1 - \alpha)(1 - \lambda_\alpha) \\
&\geq 0
\end{aligned}$$

thus $\frac{\delta p}{\delta \lambda} \leq 0$.

⁴ $F_R^*(K_3) = 1/2$ implies too that $F_R^*(K_1) = 1/2$ by the continuity of F_R^* .

The following three other proofs rely on the fact that $\lambda_\alpha \geq .5$.

- $sgn\left(\frac{\delta p_\alpha}{\delta \lambda}\right) = sgn(\alpha + 2\lambda_\alpha - 4\alpha\lambda_\alpha - 4\lambda_\alpha^2(1 - \alpha))$ and

$$\begin{aligned}
& \alpha + 2\lambda_\alpha - 4\alpha\lambda_\alpha - 4\lambda_\alpha^2(1 - \alpha) \\
&= 2\lambda_\alpha(1 - \alpha) + \alpha - 2\alpha\lambda_\alpha - 4\lambda_\alpha^2(1 - \alpha) \\
&= (1 - \alpha)(2\lambda_\alpha - 4\lambda_\alpha^2) + \alpha(1 - 2\lambda_\alpha) \\
&= (1 - \alpha)2\lambda_\alpha(1 - 2\lambda_\alpha) + \alpha(1 - 2\lambda_\alpha) \\
&\leq 0
\end{aligned}$$

Thus $\frac{\delta p_\alpha}{\delta \lambda} \leq 0$

- $sgn\left(\frac{\delta p}{\delta \lambda_\alpha}\right) = sgn(-2\lambda_\alpha^2(1 - \alpha) - \alpha + \lambda)$ and

$$\begin{aligned}
& -2\lambda_\alpha^2(1 - \alpha) - \alpha + \lambda \\
&\leq -\lambda_\alpha(1 - \alpha) - \alpha + \lambda \\
&= \alpha(\lambda_\alpha - 1) + \lambda - \lambda_\alpha \\
&\leq 0
\end{aligned}$$

Thus $\frac{\delta p}{\delta \lambda_\alpha} \leq 0$.

- $sgn\left(\frac{\delta p_\alpha}{\delta \lambda_\alpha}\right) = sgn(4\lambda_\alpha^2(1 - \alpha) - (-\alpha + \lambda))$ and

$$4\lambda_\alpha^2(1 - \alpha) - (-\alpha + \lambda) \geq 1 - \alpha + \alpha - \lambda = 1 - \lambda \geq 0.$$

Thus $\frac{\delta p_\alpha}{\delta \lambda_\alpha} \leq 0$.

8.3 Proof of proposition 3

The proof proceed in three steps: 1) we first show that the voters' expected welfare under the scenario of Lizzeri and Persico (2001) is greater than that under our scenario when $\lambda = \lambda_\alpha$ whenever α is smaller than some threshold; 2) we then show that the electorate's expected welfare is higher under our scenario when $\lambda < 1 = \lambda_\alpha$, 3) finally, we show that the voters' welfare is (weakly) increasing in λ_α .

1. Compute

$$EU(CS) = p\lambda + 1 - p - p_\alpha + p_\alpha[(1 - \alpha)\lambda_\alpha + \alpha]$$

when $\lambda = \lambda_\alpha$. It is immediate to check then that $EU(LP01) > EU(CS)$ if and only if

$$\alpha < \frac{\lambda - \sqrt{\lambda(1-\lambda)}}{2\lambda - 1}.$$

2. Next, compute $EU(CS)$ when $\lambda_\alpha = 1$. It is easy to verify that $EU(CS) = 1$ for any value of λ whereas $EU(LP01) < 1$ for any $\lambda < 1$ and $EU(LP01) = 1$ if and only if $\lambda = 1$. Thus

$$EU(LP01)|_{\lambda_\alpha=1} \leq EU(CS)|_{\lambda_\alpha=1}.$$

3. Finally, given our comparative statics results, it is immediate to see that

$$\frac{\partial EU(CS)}{\partial \lambda_\alpha} \geq 0$$

QED

8.4 Proof of proposition (To be checked)

Suppose my opponent follows the proposed equilibrium strategy. Denote with $Sh[S(x), F(x)]$ my share of the vote when my opponent plays the equilibrium strategy $S(x)$.

We first prove that $1 - p - p_\alpha = 1/2$ and $p + p_\alpha = 1/2$. Suppose I choose full redistribution. Then I can win with probability 1 against my opponent if he decides to go for full redistribution according to the equilibrium distribution. All I have to do is pick a new continuous distribution function \tilde{F}_R such that I offer 0 to some of the voters who would have received an offer ranging between 0 and K_1 according to F_R^* and use the proceeds of this change to make more offers in the range $[K_3, 2\lambda]$. Yet, doing this implies that I lose with probability 1 if my opponent plays any of the other two strategies. Indeed, whenever I follow the above deviation we have $\tilde{F}_R(K_2) < 1/2$ and $\tilde{F}_R(1) < 1/2$ whereas $F_\alpha^*(K_2) = 1/2$ and, slightly abusing notation, $F_{No\ taxes}^*(1) = 1$ - given that $K_1 < K_2 < 1 < K_3$. As a consequence, deviating implies:

$$Sh[F_\alpha^*(K_2), \tilde{F}_R(K_2)] < 1/2$$

and

$$Sh[F_{No\ taxes}^*(1), \tilde{F}_R(1)] < 1/2.$$

Thus, if my opponent plays full redistribution with a probability lower (resp. higher) than $1/2$, I get a payoff that is lower (resp. higher) than $1/2$, which is a contradiction with respect to the required equilibrium probability of the game, $1/2$. In equilibrium it must therefore be that $1 - p - p_\alpha = 1/2$ and $p + p_\alpha = 1/2$.

Writing my expected vote share against redistribution when the opponent follows the equilibrium strategy yields immediately that the bounds of the distribution followed in equilibrium are indeed K_1 and K_3 .

Finally, the fact that the distribution is uniform is due to the fact that there are two players and that the equilibrium probability of winning a vote with an offer worth x is piecewise linear in the offers, as shown by Lizzeri and Persico (2005).

Now that we know that $1 - p - p_\alpha = 1/2$, we can find the equilibrium values of p and p_α by focusing on the two other strategies only. This implies that $p = p_\alpha = 1/4$ given that the game without the option of full redistribution is equivalent to that analyzed by Lizzeri and Persico (2001). This also allows us to prove that the intermediate bound for partial redistribution is indeed K_2 . QED