

Embracing Liberalism for Collective Identity Determination*

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Abstract

A Collective Identity Function (CIF) is a rule which aggregates personal opinions on whether an individual belongs to a certain identity into a social decision. A CIF is qualified as “simple” whenever it can be expressed in terms of winning coalitions. We characterize the class of simple CIFs in terms of three axioms, namely independence, monotonicity and self-duality. We also explore the effect of imposing conditions that ensure the equal treatment of individuals as voters or as outcomes. We show that liberalism arises as the unique simple CIF that satisfies axioms which are very natural in the collective identity determination context.

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1 Introduction

A first attempt to analyze the collective identity determination problem through concepts of social choice theory is made by Kasher and Rubinstein (1997) who, based on an exploration of Kasher (1993) about the Jewish identity, propose to treat the question as an aggregation problem. They consider a society and some abstract concept of identity (such as “being a J ”) to which every member of the society may or may not belong. Each individual has a personal opinion about whom does and whom does not belong to this identity. The collective decision is made by the aggregation of individual opinions - hence the introduction of a *collective identity function* (CIF), which maps individual opinions into a social opinion. The model, while mathematically simple, incorporates a plethora of concepts related to collective identity determination. So, leaving the modesty of its founders aside,¹ it paved the way to a growing literature, the pivots of which will be mentioned as a start for our analysis.

Liberalism appears as a central concept. Under the liberal CIF, an individual is socially conceived as belonging to some identity J if and only if he/she believes “to carry identity J ” -or “to be a J ”, so to speak. A first axiomatic characterization of liberalism is given by Kasher and Rubinstein (1997).² More recently, Samet and Schmeidler (2003) axiomatically characterize a class of CIFs which they call *consent rules*. This class is parametrized by the weights given to individuals in determining their own identity. It contains liberalism at one extreme and majoritarianism³ at the other.

Another strand of the literature views a CIF as a recursive procedure. For example, the *procedural CIF* of Kasher (1993) suggests to determine an initial set $J(0)$ of individuals who are unanimously agreed to carry identity J . All individuals who are considered to be a J by at least one member of $J(0)$ are added to $J(0)$, hence expanding the set of J s to $J(1)$. The procedure continues inductively until the set of J s cannot be expanded anymore. A variant of this procedure, where the initial set $J(0)$ consists of individuals who consider themselves as J s, is defined by Dimitrov, Sung, and Xu (2003) who characterize both procedures. As we discuss in the next section, the procedural view of CIFs is almost orthogonal to their conception as consent rules which lie between liberalism and majoritarianism.

¹Kasher and Rubinstein (1997) present it as a “purely logical exercise”

²while Dimitrov and Sung (2003) show that the five axioms used by Kasher and Rubinstein (1997) are logically dependent whereas three of them suffice to establish the desired equivalence.

³where, as also exemplified by Kasher and Rubinstein (1997), personal opinions about the identity of an individual are aggregated according to the majority rule.

We propose to approach the collective identity determination problem from a perspective where CIFs can be expressed in terms of *winning coalitions*.⁴ We qualify such CIFs as *simple*. Under a simple CIF, the information about the social opinion contained in the set of winning coalitions is the same as that in the corresponding aggregation rule. In other words, simple CIFs can be examined through their winning coalitions, which brings us a new perspective in the exploration of the collective identity determination problem.⁵

We characterize simple CIFs in terms of three axioms, namely self-duality, monotonicity and independence. Among these, self-duality imposes a certain kind of neutrality towards the identity to be aggregated: While aggregating individual opinions about who is a J , we simultaneously aggregate the individual opinions about who is not a J . Self-duality requires a symmetric treatment of these two aggregations. To be more precise, self-duality requires that in case all individuals reverse their opinions about who is a J , the social decision must also be reversed. Monotonicity says that additional opinions which consider some individual as a J cannot end up in the disqualification of this individual while he/she was previously qualified as a J . Finally, independence requires that the decision about whether a member of the society is a J or not must be made according to the opinions of the individuals about this particular member.⁶

In Section 2, we introduce and characterize simple CIFs. We also consider various CIFs of the literature. Among other things, we show that the set of simple CIFs and the set of consent rules have a non-empty intersection - a class to which we refer as *symmetric consent rules*. Here “symmetry” refers to the fact that the weight of a given individual in qualifying or disqualifying him/herself as a J is the same. We also establish a general incompatibility between procedural and simple CIFs by showing that all procedural CIFs fail self-duality and independence. Section 3 is devoted to the equal treatment of individuals as voters and as outcomes. In the usual social choice framework where voters and outcomes form disjoint sets, this is a naturally arising distinction: The equal treatment of voters is usually called “anonymity” while the equal treatment of outcomes is typically referred to as “neutrality”. On

⁴As usual, we say that a coalition K of individuals is winning for individual i if and only if the members of K , independent of the rest of the society, are able to determine whether i carries identity J or not.

⁵A detailed discussion of simple social choice rules in a general social choice setting can be found in Austen-Smith and Banks (1999).

⁶ Among the three axioms we use, self-duality and independence are as in Samet and Schmeidler (2003) who also use a monotonicity condition which is stronger than the one we introduce. However, the two monotonicities coincide over CIFs that satisfy independence.

the other hand, in our environment where voters and outcomes coincide, we need a more subtle discrimination. So we adapt the standard anonymity and neutrality conditions of social choice theory to our model. We define an anonymous CIF as one which decides on the identity of an individual i by equally treating all opinions but possibly the opinion of i about him/herself.⁷ Similarly, a neutral CIF decides on the identities of any two individuals i and j by the same rule whenever voters and outcomes are disjoint, i.e., at profiles where a winning coalition over i contains neither i nor j . We characterize the class of simple, anonymous and neutral CIFs in terms of symmetric consent rules. This result, combined with the Samet and Schmeidler (2003) characterization of symmetric consent rules in terms of self-duality, monotonicity, independence and symmetry shows that over the domain of simple CIFs, the anonymity and neutrality conditions we propose successfully decompose the symmetry condition of Samet and Schmeidler (2003) which is an incorporation of both equal treatment properties. Section 4 is devoted to rights of self-determination and contains various characterizations of liberalism with reference to our previous results. Here we show that the liberal CIF arises as the unique simple CIF that satisfies axioms which are very natural in the collective identity determination context. Section 5 concludes.

2 Simple Collective Identity Functions

We consider a society N which is a finite set of individuals with $\#N = n \geq 2$. The society is confronted to the problem of deciding on its members who belong to some “group” or who carry a certain given identity.⁸ For each $i \in N$, we write $G_i \subseteq N$ for the set of individuals whom i perceives as a member of the group. We refer to G_i as the *opinion* of i . An (*opinion*) *profile* is an n -tuple $(G_1, \dots, G_n) \in \Gamma$ where $\Gamma = (2^N)^n$ is the set of all profiles. A *Collective Identity Function* (CIF) is a mapping $F : \Gamma \rightarrow 2^N$ that assigns a subset of individuals to each profile.

For each $i \in N$, we define some family $\omega(i) \subseteq 2^N$ of subsets of N such that

- (i) For each $K \in 2^N$, we have either $K \in \omega(i)$ or $N \setminus K \in \omega(i)$.
- (ii) For each $K, K' \in 2^N$ with $K \in \omega(i)$ and $K' \supseteq K$, we have $K' \in \omega(i)$

We refer to $\omega(i)$ as the set of *winning coalitions* over i .

⁷de Andrés and García-Lapresta (2004) introduce the same anonymity condition in a more general framework where individuals’ non-dichotomous opinions belong to the 0 -1 interval.

⁸or “who are a J ”, to use the original terminology.

Remark 2.1 As an implication of conditions (i) and (ii) that we impose over winning coalitions, for each $i \in N$, we have $N \in \omega(i)$ hence $\emptyset \notin \omega(i)$. Moreover, given any $K, L \in 2^N$ we have $K, L \in \omega(i) \implies K \cap L \neq \emptyset$.

A collection $\{\omega(i)\}_{i \in N}$ of winning coalitions induces a (unique) CIF $F : \Gamma \rightarrow 2^N$ in the following natural way: Given any $G \in \Gamma$ and any $i \in N$, we have $i \in F(G) \iff \{j \in N : i \in G_j\} \in \omega(i)$. We qualify a CIF $F : \Gamma \rightarrow 2^N$ as *simple* if and only if F is induced by a collection $\{\omega(i)\}_{i \in N}$ of winning coalitions.

Simple CIFs can be characterized in terms of the following three logically independent axioms:

Monotonicity (MON): A CIF $F : \Gamma \rightarrow 2^N$ is said to be *monotonic* if and only if given any $i \in N$ and any two profiles $G, G' \in \Gamma$ such that

1. $G_j = G'_j$ or $G_j = G'_j \cup \{i\}$ for all $j \in N$

and

2. $\exists k \in N$ such that $i \notin G'_k$ but $G_k = G'_k \cup \{i\}$

we have $i \in F(G') \implies i \in F(G)$.

Independence (I): A CIF $F : \Gamma \rightarrow 2^N$ is said to be *independent* if and only if given any $i \in N$ and any $G, G' \in \Gamma$ with $i \in G_j \iff i \in G'_j$ for all $j \in N$, we have $i \in F(G) \iff i \in F(G')$.

Self-Duality (SD): A CIF $F : \Gamma \rightarrow 2^N$ is said to satisfy *self-duality* if and only if given any $G, \bar{G} \in \Gamma$ such that $\bar{G}_i = N \setminus G_i$ for all $i \in N$, we have $F(\bar{G}) = N \setminus F(G)$.

Samet and Schmeidler (2003) establish the logical independence of self-duality, independence and the monotonicity condition they use. As their and our monotonicities coincide for independent CIFs, we are able to borrow their example which shows that the conjunction of independence and monotonicity does not imply self-duality.⁹ The CIF F which is defined for each $G \in \Gamma$ as $F(G) = N \setminus \{i \in N : i \in G_i\}$ satisfies self-duality and independence but not monotonicity. Finally, to see that the conjunction of self-duality and monotonicity does not imply independence, consider the CIF F defined as follows: At each $G \in \Gamma$ and for all $i \in N$,

- if $i \in G_i$, then $i \in F(G) \iff \#\{j \in N \setminus \{i\} : j \in G_j\} \leq \frac{n-1}{2}$
- if $i \notin G_i$, then $i \notin F(G) \iff \#\{j \in N \setminus \{i\} : j \notin G_j\} \leq \frac{n-1}{2}$

⁹See Footnote 6.

Theorem 2.1 *A CIF $F : \Gamma \rightarrow 2^N$ is simple if and only if F satisfies MON, SD and I.*

Proof. To prove the “only if” part, take any simple CIF F and let $\{\omega(i)\}_{i \in N}$ be the family of winning coalitions which induces F . To show that F satisfies MON, take any $i \in N$ and any $G, G' \in \Gamma$ such that $G_j = G'_j$ or $G_j = G'_j \cup \{i\}$ for all $j \in N$ while $i \notin G'_k$ but $G_k = G'_k \cup \{i\}$ for some $k \in N$. Let $i \in F(G')$. As F is simple, we have $\{j \in N : i \in G'_j\} \in \omega(i)$. Moreover $\{j \in N : i \in G_j\} \supset \{j \in N : i \in G'_j\}$. Thus, by the definition of $\omega(i)$, we have $\{j \in N : i \in G_j\} \in \omega(i)$, which implies $i \in F(G)$, establishing the monotonicity of F . To show that F satisfies I, take any $i \in N$ and any $G, G' \in \Gamma$ with $i \in G_j \iff i \in G'_j$ for all $j \in N$. Hence $\{j \in N : i \in G_j\} = \{j \in N : i \in G'_j\}$. Suppose for a contradiction that $i \in F(G)$ but $i \notin F(G')$. As $i \in F(G)$, we have $\{j \in N : i \in G_j\} \in \omega(i)$, so $\{j \in N : i \in G'_j\} \in \omega(i)$. But this implies that $i \in F(G')$ which contradicts with $i \notin F(G')$, establishing the independence of F . To show that F satisfies SD, take any $G, \bar{G} \in \Gamma$ such that $\bar{G}_i = N \setminus G_i$ for all $i \in N$. Take any $j \in F(G)$ and assume for a contradiction $j \in F(\bar{G})$. As $j \in F(G)$ we have $\{i \in N : j \in G_i\} \in \omega(j)$ and as $j \in F(\bar{G})$, we have $\{i \in N : j \in \bar{G}_i\} \in \omega(j)$. However, by the choice of G and \bar{G} , $\{i \in N : j \in G_i\}$ and $\{i \in N : j \in \bar{G}_i\}$ are disjoint, which contradicts that $\omega(j)$ is a family of winning coalitions. Taking any $j \notin F(G)$ and assuming $j \notin F(\bar{G})$ leads to a similar contradiction, showing that F satisfies SD.

To prove the “if” part, take any CIF F which satisfies MON, SD and I. For each $i \in N$, define a set $\gamma(i) \subseteq 2^N$ as follows: For any $K \in 2^N$, take some $G \in \Gamma$ such that $i \in G_k$ for all $k \in K$ and $i \notin G_k$ for all $k \in N \setminus K$. Let $K \in \gamma(i) \iff i \in F(G)$. As F satisfies independence, the choice of G does not matter. Thus $\gamma(i)$ is well-defined. We will complete the proof by showing that $\{\gamma(i)\}_{i \in N}$ is a collection of winning coalitions that induces F . To see this, we first establish that for any $K \in 2^N$, we have either $K \in \gamma(i)$ or $N \setminus K \in \gamma(i)$. Assume, without loss of generality, that $K \in \gamma(i)$. Take some $G \in \Gamma$ with $i \in G_k$ for all $k \in K$ and $i \notin G_k$ for all $k \in N \setminus K$. By the definition of $\gamma(i)$ we have $i \in F(G)$. Now consider $\bar{G} \in \Gamma$ where $\bar{G}_i = N \setminus G_i$ for all $i \in N$. By SD, we have $i \notin F(\bar{G})$. But note that $i \in \bar{G}_k$ for all $k \in N \setminus K$ and $i \notin \bar{G}_k$ for all $k \in K$ which, by I, implies $N \setminus K \notin \gamma(i)$. We will now establish that for any $K, K' \in 2^N$ with $K' \in \gamma(i)$ and $K \supseteq K'$, we have $K \in \gamma(i)$. As $K' \in \gamma(i)$, there exists some $G' \in \Gamma$ with $i \in G'_k$ for all $k \in K'$ and $i \notin G'_k$ for all $k \in N \setminus K'$ while $i \in F(G')$. Now consider $G \in \Gamma$ which is defined as $G_j = G'_j \cup \{i\}$ for all $j \in K \setminus K'$ and $G_j = G'_j$ for all $j \in (N \setminus K) \cup K'$. As F satisfies MON we have $i \in F(G)$ which, by the

definition of $\gamma(i)$ and I, implies $K \in \gamma(i)$. So $\{\gamma(i)\}_{i \in N}$ is a collection of winning coalitions which, by its very definition, induces F . ■

We now extend our analysis to certain CIFs of the literature:

1. The *liberal CIF* $L : \Gamma \rightarrow 2^N$ introduced by Kasher and Rubinstein (1997) is defined for each $G \in \Gamma$ as $L(G) = \{i \in N : i \in G_i\}$. L satisfies MON, SD and I - hence by Theorem 2.1 is a simple CIF. In fact, its corresponding collection of winning coalitions is defined for each $i \in N$ as $\omega(i) = \{K \in 2^N : i \in K\}$.
2. The *consensus CIF* $C : \Gamma \rightarrow 2^N$ which can be found in Kasher and Rubinstein (1997) is defined for each $G \in \Gamma$ as $C(G) = \{i \in N : i \in G_j \text{ for all } j \in N\}$. Although C satisfies MON and I, it fails SD - hence by Theorem 2.1 is not a simple CIF. So it cannot be written as a collection of winning coalitions.
3. The *dictatorial CIF* $F_d : \Gamma \rightarrow 2^N$ (where some $d \in N$ is the dictator) which can also be found in Kasher and Rubinstein (1997) is defined for each $G \in \Gamma$ as $F_d(G) = G_d$. F_d satisfies MON, SD and I - hence by Theorem 2.1 is a simple CIF. Its corresponding collection of winning coalitions is defined for each $i \in N$ as $\omega(i) = \{K \in 2^N : d \in K\}$.
4. Let n be odd. The *majoritarian CIF* $M : \Gamma \rightarrow 2^N$ which can be found in Samet and Schmeidler (2003) is defined as follows: For each $G \in \Gamma$ and each $i \in N$ we have $i \in M(G)$ if and only if $\#\{j \in N : i \in G_j\} > \frac{n}{2}$. Again M satisfies MON, SD and I - hence by Theorem 2.1 is a simple CIF. Its corresponding collection of winning coalitions is defined for each $i \in N$ as $\omega(i) = \{K \in 2^N : \#K > \frac{n}{2}\}$.
5. Procedural CIFs: Take any CIF $F^0 : \Gamma \rightarrow 2^N$. For any $G \in \Gamma$ and any non-negative integer k let $F^{k+1}(G) = F^k(G) \cup \{j \in N : j \in G_i \text{ for some } i \in F^k(G)\}$. Let \bar{k} be the smallest integer for which $F^{\bar{k}+1}(G) = F^{\bar{k}}(G)$. Define the CIF $\bar{F}^0 : \Gamma \rightarrow 2^N$ as $\bar{F}^0(G) = F^{\bar{k}}(G)$ for each $G \in \Gamma$. We call \bar{F}^0 the *procedural CIF based on* F^0 . The consensus-start-respecting procedure proposed by Kasher (1993) and the liberal-start-respecting procedure proposed by Dimitrov, Sung, and Xu (2003) are particular procedural CIFs based on the consensus CIF C and the liberal CIF L respectively. The consensus-start-respecting procedure, which is based on a non-simple CIF, fails independence and self-duality. But this is also the case for the liberal-start-respecting procedure which is based on a simple CIF. In fact, this incompatibility between procedural and simple CIFs is more general, as announced by the following proposition:

Proposition 2.1 *Take any simple CIF $F^0 : \Gamma \rightarrow 2^N$. The procedural CIF \overline{F}^0 based on F^0 fails independence and self-duality.*

Proof. Let F^0 and \overline{F}^0 be as in the statement of the proposition. Let $\{\omega(i)\}_{i \in N}$ be the family of winning coalitions of the simple CIF F^0 . We first show that \overline{F}^0 fails independence. Take any $i \in N$ and any $K \in \omega(i)$ which differs from N and $N \setminus \{i\}$. Consider the profile $G \in \Gamma$ where $G_k = N \setminus \{i\}$ for all $k \in K$ and $G_k = N$ for all $k \in N \setminus K$. So $F^0(G) = N \setminus \{i\}$ and, as K differs from $N \setminus \{i\}$ and N , $\overline{F}^0(G) = N$. Now consider the profile $G' \in \Gamma$ where $G'_k = \emptyset$ for all $k \in K$ and $G'_k = \{i\}$ for all $k \in N \setminus K$. As $N \setminus K \notin \omega(i)$, we have $F^0(G') = \emptyset = \overline{F}^0(G')$. Remark that $i \in G_k \Leftrightarrow i \in G'_k$ for all $k \in N$, while $i \in \overline{F}^0(G)$ but $i \notin \overline{F}^0(G')$, showing that \overline{F}^0 fails independence.

To see that \overline{F}^0 fails self-duality, take any $i \in N$ and any $K \in \omega(i)$ which differs from $N \setminus \{i\}$ and N . Consider the profile $G \in \Gamma$ where $G_k = N \setminus \{i\}$ for all $k \in K$ and $G_k = N$ for all $k \in N \setminus K$. So $F^0(G) = N \setminus \{i\}$ and, as K differs from $N \setminus \{i\}$ and N , $\overline{F}^0(G) = N$. Now consider the profile $G' \in \Gamma$ where $G'_k = \{i\}$ for all $k \in K$ and $G'_k = \emptyset$ for all $k \in N \setminus K$. As $K \in \omega(i)$, we have $F^0(G') = \{i\} = \overline{F}^0(G')$. Remark that $G'_k = N \setminus G_k$ for all $k \in N \setminus K$, while $i \in \overline{F}^0(G) \cap \overline{F}^0(G')$, showing that \overline{F}^0 fails self-duality. ■

6. The consent rules of Samet and Schmeidler (2003) are parametrized by two positive integers s and t with $s + t \leq n + 2$. A *consent rule (with consent quotas s and t)* is a CIF $F^{st} : \Gamma \rightarrow 2^N$ such that given any $G \in \Gamma$ and any $i \in N$

- if $i \in G_i$, then $i \in F^{st}(G) \iff \#\{j \in N : i \in G_j\} \geq s$
- if $i \notin G_i$, then $i \notin F^{st}(G) \iff \#\{j \in N : i \notin G_j\} \geq t$

Taking $s = t$ is a case of particular interest where we call F^{st} a *symmetric consent rule (with quota s)* and denote it F^s . Remark that for symmetric consent rules, the quota varies between $s = 1$ and $s = \lfloor \frac{n}{2} \rfloor + 1$.¹⁰ At one extreme where $s = 1$, F^s coincides with the liberal CIF L . At the other extreme where $s = \lfloor \frac{n}{2} \rfloor + 1$, we go to majoritarianism.¹¹

¹⁰We write $\lfloor n/2 \rfloor$ for the highest integer less than or equal to $n/2$.

¹¹This has two subcases which is worth distinguishing. When n is odd, $F^{\lfloor n/2 \rfloor + 1}$ coincides with the majoritarian CIF M . When n is even, we have two versions of majoritarianism depending on the choice of $s \in \{\frac{n}{2}, \frac{n}{2} + 1\}$. When $s = n/2$, a coalition K of cardinality $n/2$ is winning over an individual i if and only if i is a member of K . On the other hand, when $s = \frac{n}{2} + 1$, a coalition K of cardinality $n/2$ is winning over an individual i if and only if i is not a member of K .

Not every consent rule is simple. In fact, the intersection of the set of consent rules with the set of simple CIFs is the set of symmetric consent rules - a result which we formally state in the following proposition:

Proposition 2.2 *A consent rule $F^{st} : \Gamma \rightarrow 2^N$ is a simple CIF if and only if F^{st} is a symmetric consent rule, i.e., $s = t$.*

Proof. To prove the “only if” part, we refer to Proposition 2 of Samet and Schmeidler (2003) which establishes that a consent rule satisfies self-duality if and only if it is a symmetric consent rule. This result, combined with our Theorem 2.1, implies that a consent rule is a simple CIF only if it is a symmetric consent rule. To show the “if” part, we check that symmetric consent rules satisfy MON, SD and I^{12} -hence are simple CIFs by Theorem 2.1. ■

Remark 2.2 *When n is odd while $s \in \{1, \dots, \lfloor \frac{n}{2} \rfloor + 1\}$ or n is even while $s \in \{1, \dots, \frac{n}{2}\}$, the winning coalitions of the symmetric consent rule F^s are defined for every $i \in N$ as $\omega(i) = \{K \in 2^N : i \in K \text{ and } s \leq \#K \leq n - s\} \cup \{K \in 2^N : \#K > n - s\}$. So $\omega(i)$ consists of coalitions*

- whose cardinality varies from s to $n - s$ while they contain i ¹³
- whose cardinality exceeds $n - s$ (independent of whether they contain i or not).

On the other hand, when n is even and $s = \frac{n}{2} + 1$, we have $\omega(i) = \{K \in 2^N : i \notin K \text{ and } \#K = n/2\} \cup \{K \in 2^N : \#K > \frac{n}{2}\}$ for every $i \in N$.

3 Equal Treatment of Individuals

The literature of social choice theory contains two well-known equal treatment conditions, one for voters (usually called “anonymity”) and one for outcomes (usually called “neutrality”). In a framework where voters and alternatives form mutually exclusive sets, the conceptual discrimination between these two conditions is straightforward. On the other hand, the matter is more complicated to handle when voters and outcomes coincide - as is the case in our model.

Samet and Schmeidler (2003) by-pass the problem by introducing a “symmetry” condition which incorporates both kinds of equal treatment conditions.¹⁴ Given a bijection $\Pi : N \rightarrow N$ over N , we write, by a slight abuse

¹²A result which is also established by Theorem 2 of Samet and Schmeidler (2003).

¹³Remark that there is no such coalition when n is odd and $s = \lfloor \frac{n}{2} \rfloor + 1$.

¹⁴See Section 4.1 of Samet and Schmeidler (2003) where they discuss their symmetry axiom.

of notation, $\Pi(K) = \{\Pi(j) : j \in K\}$ for any non-empty $K \subseteq N$. By a more considerable abuse of notation, for any $G \in \Gamma$, we mean by $\Pi(G)$ a profile G' such that $G'_{\Pi(j)} = G_j$ for each $j \in N$. We say that a CIF $F : \Gamma \rightarrow 2^N$ is *Samet-Schmeidler symmetric* if and only if given any permutation $\Pi : N \rightarrow N$, any $G \in \Gamma$ and any $i \in N$, we have $i \in F(G) \Leftrightarrow \Pi(i) \in F(\Pi(G))$.

Remark 3.1 *For a simple CIF F , Samet-Schmeidler symmetry imposes a particular structure over the family of winning coalitions $\{\omega(i)\}_{i \in N}$ so that given any $\Pi : N \rightarrow N$, any $i \in N$ and any $K \in 2^N$, we have $K \in \omega(i) \Leftrightarrow \Pi(K) \in \omega(\Pi(i))$.*

It is possible to extract from Samet-Schmeidler symmetry, a voters' equal treatment property which requires that while deciding whether some individual $i \in N$ is a J , all individuals, with the possible exception of i him/herself, must be equally treated. Fixing some $i \in N$ and letting $\Pi^{-i} : N \rightarrow N$ stand for some bijection with $\Pi^{-i}(i) = i$, we say that a CIF $F : \Gamma \rightarrow 2^N$ is *anonymous* if and only if for all $i \in N$ and for all $G \in \Gamma$, we have $i \in F(G) \Leftrightarrow \Pi^{-i}(i) = i \in F(\Pi^{-i}(G))$. Clearly, Samet-Schmeidler symmetry implies anonymity.¹⁵

Remark 3.2 *For a simple CIF F , anonymity imposes a particular structure over the family of winning coalitions $\{\omega(i)\}_{i \in N}$ so that for each $i \in N$ and for all $K \in 2^N$, $K \in \omega(i) \Leftrightarrow \Pi^{-i}(K) \in \omega(i)$. In other words, a coalition K which does not contain individual i is winning over i if and only if every coalition K' with $\#K' = \#K$ and which does not contain i is winning over i as well. Similarly, a coalition K which contains i is winning over i if and only if every coalition K' with $\#K' = \#K$ and which contains i is winning over i as well.*

Anonymous simple CIFs can be characterized in terms of what we call generalized symmetric consent rules. Fix some n -tuple of positive integers $\bar{s} = (s_1, \dots, s_n)$ such that $s_i \in \{1, \dots, \lfloor \frac{n}{2} \rfloor + 1\}$ for each $i \in N$. A *generalized symmetric consent rule (with quota \bar{s})* is a CIF $F^{\bar{s}} : \Gamma \rightarrow 2^N$ such that given any $G \in \Gamma$ and any $i \in N$

if $i \in G_i$, then $i \in F^{\bar{s}}(G) \iff \#\{j \in N : i \in G_j\} \geq s_i$

if $i \notin G_i$, then $i \notin F^{\bar{s}}(G) \iff \#\{j \in N : i \notin G_j\} \geq s_i$

Note that symmetric consent rules of Samet and Schmeidler (2003) are particular cases of $F^{\bar{s}}$ where \bar{s} is such that $s_i = s_j$ for all $i, j \in N$.¹⁶

¹⁵while the converse implication does not hold which we show through an example in Footnote 16

¹⁶In fact, a generalized symmetric consent rules $F^{\bar{s}}$ where $s_i \neq s_j$ for some $i, j \in N$ is an example of a CIF which is anonymous but not Samet-Schmeidler symmetric. See Footnote 15.

Theorem 3.1 *A CIF $F : \Gamma \rightarrow 2^N$ is simple and anonymous if and only if F is a generalized symmetric consent rule $F^{\bar{s}}$ with $s_i \in \{1, \dots, \lfloor \frac{n}{2} \rfloor + 1\}$ for each $i \in N$.*

Proof. We leave the “if” part to the reader by asking to check that any generalized symmetric consent rule $F^{\bar{s}}$ satisfies anonymity, MON, SD and I, which, by Theorem 2.1, implies that $F^{\bar{s}}$ is anonymous and simple. To show the “only if” part, take any simple and anonymous CIF $F : \Gamma \rightarrow 2^N$ with a family of winning coalitions $\{\omega(i)\}_{i \in N}$. For each $i \in N$, let $s_i = \min \{\#K : K \in \omega(i)\}$. First consider the case where n is odd. As, by the definition of a winning coalition, either $K \in \omega(i)$ or $N \setminus K \in \omega(i)$ holds for each $K \in 2^N$, we have $s_i \in \{1, \dots, \lfloor \frac{n}{2} \rfloor + 1\}$. Moreover, there exists $\bar{K} \in \omega(i)$ with $i \in \bar{K}$ and $\#\bar{K} = s_i$. To see this, suppose the contrary. In case $s_i \in \{1, \dots, \lfloor \frac{n}{2} \rfloor\}$, there exists, by the anonymity of F , $K, K' \in \omega(i)$ such that $\#K = \#K' = s_i$ while $K \cap K' = \emptyset$, which contradicts the definition of a winning coalition. In case $s_i = \lfloor \frac{n}{2} \rfloor + 1$, there exists, by the definition of a winning coalition, $K \in \omega(i)$ with $i \notin K$ and $\#K = \lfloor \frac{n}{2} \rfloor$, which contradicts the choice of $s_i = \min \{\#K : K \in \omega(i)\}$. Now, as there exists $\bar{K} \in \omega(i)$ with $i \in \bar{K}$ and $\#\bar{K} = s_i$, by the anonymity of F , we have $K \in \omega(i)$ for any $K \in 2^N$ with $i \in K$ and $\#K = s_i$. Moreover, as F satisfies MON, we have $K \in \omega(i)$ for any $K \in 2^N$ with $i \in K$ and $\#K \geq s_i$. So, $\{K \in 2^N : i \in K \text{ and } \#K \geq s_i\} \subseteq \omega(i)$. As $s_i = \min \{\#K : K \in \omega(i)\}$, by the definition of a winning coalition we have $\{K \in 2^N : \#K > n - s_i\} \subseteq \omega(i)$. By the fact that either $K \in \omega(i)$ or $N \setminus K \in \omega(i)$ holds for each $K \in 2^N$, we have $\omega(i) = \{K \in 2^N : i \in K \text{ and } \#K \geq s_i\} \cup \{K \in 2^N : \#K > n - s_i\}$. So, we conclude, by referring to Remark 2.2, that the decision over i is taken according to a symmetric consent rule with quota s_i .

Now consider the case where n is even. As for each $K \in 2^N$, either $K \in \omega(i)$ or $N \setminus K \in \omega(i)$ holds, we have $s_i \in \{1, \dots, \frac{n}{2}\}$. In case $s_i \neq \frac{n}{2}$, the arguments we used for the case where n is odd show the existence of $\bar{K} \in \omega(i)$ with $i \in \bar{K}$ and $\#\bar{K} = s_i$ and the anonymity and monotonicity of F similarly establishes that the decision over i is taken according to a symmetric consent rule with quota s_i . In case $s_i = \frac{n}{2}$, we have, by the anonymity of F and the definition of a winning coalition, two possible mutually exclusive cases:

CASE 1: $K \in \omega(i)$ for all $K \in 2^N$ with $i \in K$ and $\#K = \frac{n}{2}$ while $K \notin \omega(i)$ for all $K \in 2^N$ with $i \notin K$ and $\#K = \frac{n}{2}$.

CASE 2: $K \in \omega(i)$ for all $K \in 2^N$ with $i \notin K$ and $\#K = \frac{n}{2}$ while $K \notin \omega(i)$ for all $K \in 2^N$ with $i \in K$ and $\#K = \frac{n}{2}$.

For CASE 1, by the monotonicity of F , we have $\omega(i) = \{K \in 2^N : i \in K \text{ and } \#K = s_i\} \cup \{K \in 2^N : \#K > n - s_i\}$. So we conclude, by referring to Remark 2.2, that the decision over i is taken according to a symmetric

consent rule with quota $s_i = \frac{n}{2}$. For CASE 2, by the monotonicity of F , we have $\omega(i) = \{K \in 2^N : i \notin K \text{ and } \#K = n/2\} \cup \{K \in 2^N : \#K > \frac{n}{2}\}$. So we conclude, by referring to Remark 2.2, that the decision over i is taken according to a symmetric consent rule with quota $s_i + 1 = \frac{n}{2} + 1$. ■

Remark that while deciding on the identity of some $i \in N$, anonymity does not bring any restriction on the decision power of i compared to the (equal) decision powers of the other individuals. In other words, under anonymity, while all individuals but i are equally treated as voters, the opinion of i about him/herself may be favored, disfavored or equally treated compared to the other individuals' opinions over i . The following three conditions classify the set of simple and anonymous CIFs according to this notion:

SF^+ : A simple and anonymous CIF $F : \Gamma \rightarrow 2^N$ with a family of winning coalitions $\{\omega(i)\}_{i \in N}$ is *self-favoring* for $i \in N$ if and only if

- there exists $K \in 2^N$ with $i \notin K$ and $K \notin \omega(i)$ such that given any $j \in K$ we have $(K \setminus \{j\}) \cup \{i\} \in \omega(i)$.
- for all $K \in 2^N$ with $i \notin K$, we have $K \in \omega(i) \implies (K \setminus \{j\}) \cup \{i\} \in \omega(i)$ for all $j \in K$

SF^- : A simple CIF $F : \Gamma \rightarrow 2^N$ with a family of winning coalitions $\{\omega(i)\}_{i \in N}$ is *self-disfavoring* for $i \in N$ if and only if

- there exists $K \in 2^N$ with $i \in K$ and $K \notin \omega(i)$ such that given any $j \in N \setminus K$ we have $(K \setminus \{i\}) \cup \{j\} \in \omega(i)$.
- for all $K \in 2^N$ with $i \in K$, we have $K \in \omega(i) \implies (K \setminus \{i\}) \cup \{j\} \in \omega(i)$ for all $j \in N \setminus K$

SF^0 : A simple CIF $F : \Gamma \rightarrow 2^N$ with a family of winning coalitions $\{\omega(i)\}_{i \in N}$ is *self-disregarding* for $i \in N$ if and only if given any $K \in 2^N$ with $i \in K$ and any $j \in N \setminus K$, we have $K \in \omega(i) \Leftrightarrow (K \setminus \{i\}) \cup \{j\} \in \omega(i)$.

This treatment of self-opinions by generalized symmetric consent rules depends on the quota s_i and the number of individuals n in the society, as we remark below:

Remark 3.3 A generalized symmetric consent rule $F^{\bar{s}} : \Gamma \rightarrow 2^N$ is

- (i) *self-favoring* for $i \in N$ if and only if $s_i \in \{1, \dots, \lfloor \frac{n}{2} \rfloor\}$
- (ii) *self-disfavoring* for $i \in N$ if and only if n is even and $s_i = \frac{n}{2} + 1$
- (iii) *self-disregarding* for $i \in N$ if and only if n is odd and $s_i = \lfloor \frac{n}{2} \rfloor + 1$

Observe that almost all generalized symmetric consent rules are self-favoring, except two cases: When n is even, it is self-disfavoring to determine the identity of i by a version of majoritarianism where a coalition K of cardinality $n/2$ is winning over an individual i if and only if i is not a member of K .¹⁷ Similarly, when n is odd, determining individual identities by (usual) majoritarianism is self-disregarding.

We now turn to the equal treatment of individuals as outcomes. Recall that Samet-Schmeidler symmetry is pretended to reflect the equal treatment property both for voters and outcomes. Our anonymity condition extracts the former part of this. Hence, we allow ourselves to say that a CIF $F : \Gamma \rightarrow 2^N$ is *Samet-Schmeidler neutral* if and only if given any permutation $\Pi : N \rightarrow N$ with $\Pi(i) \neq i$, any $G \in \Gamma$ and any $i \in N$, we have $i \in F(G) \Leftrightarrow \Pi(i) \in F(\Pi(G))$. Samet-Schmeidler neutrality is quite a demanding condition. In fact, it is equivalent to Samet-Schmeidler symmetry.¹⁸ Moreover, dictatorial CIFs, which are perfectly consistent with the idea of using the same decision rule for all individuals, fail to satisfy it.¹⁹ Not only this clashes with the standard connotation of neutrality and dictatorship in social choice theory, but it also seems to impose a structure more than necessary to ensure that “the same rule is used by society to determine the qualification of each individual”.²⁰

Thus, we look for a less demanding neutrality condition which ensures the equal treatment of individuals as outcomes while it is congruous to our model as well as to the usual connotations of social choice theory. The complication of the matter arises from the fact that voters and outcomes coincide. So we propose to impose the usual neutrality requirement only for cases where voters and outcomes differ. We say that a CIF $F : \Gamma \rightarrow 2^N$ is *essentially neutral* if and only if given any $i, j \in N$, the existence of some

¹⁷See footnote 11

¹⁸Samet-Schmeidler symmetry, by definition, implies Samet-Schmeidler neutrality. To see the converse implication, define for each $i, j \in N$, a bijection $\Pi^{ij} : N \rightarrow N$ as $\Pi^{ij}(i) = j$, $\Pi^{ij}(j) = i$ and $\Pi^{ij}(k) = k$ for all $k \in N \setminus \{i, j\}$. Note that any bijection over N can be expressed as the composition of some family of bijections Π^{ij} over N . Thus every CIF that is Samet-Schmeidler neutral is also Samet-Schmeidler symmetric.

¹⁹To see this, consider a society $N = \{1, 2\}$. Let F_1 be the CIF where individual 1 is the dictator. Thus, we have $\omega(1) = \omega(2) = \{\{1\}, \{1, 2\}\}$. On the other hand, given that $\{1\} \in \omega(1)$, Samet-Schmeidler neutrality requires that $\{2\} \in \omega(2)$. This requirement implicitly assumes that $\{1\} \in \omega(1)$ because the opinion of individual 1 about himself is particular - which is not the case when 1 is the dictator. To be sure, under F_1 , the opinion of individual 1 about him/herself is fully decisive. However, this is a result of the fact that the CIF in question concentrates all decision power to 1 and not because that the opinion of individuals about themselves are favored.

²⁰See Samet and Schmeidler (2003), Section 4.1, p.225.

$G \in \Gamma$ with $i \notin G_i \cup G_j$ and $i \in F(G)$ implies the existence of some $G' \in \Gamma$ with $j \in G'_k \Leftrightarrow i \in G_k$ for all $k \in N$ such that $j \in F(G')$.

Remark 3.4 For simple CIFs, essential neutrality imposes a particular structure over the family of winning coalitions $\{\omega(i)\}_{i \in N}$ so that for any $i, j \in N$ and for any $K \in 2^N$ with $i, j \notin K$, we have $K \in \omega(i) \implies K \in \omega(j)$.²¹

In words, for simple CIFs, essential neutrality imposes that a coalition K which excludes some $i, j \in N$ is winning over i if and only if K is winning over j - a requirement which incorporates the usual neutrality idea to our model for cases where those who decide and those over which the decision is made form disjoint sets.

Note that even when simple CIFs are essentially neutral, two individuals may have smallest winning coalitions of different cardinalities - a fact which certainly contradicts the idea of using the “same” rule for all individuals.²² So we strengthen essential neutrality by qualifying a CIF $F : \Gamma \rightarrow 2^N$ as *neutral* whenever F is essentially neutral and given any $i, j \in N$, any $G \in \Gamma$ with $i \in F(G)$, there exists $G' \in \Gamma$ with $\#\{k \in N : j \in G'_k\} = \#\{k \in N : i \in G_k\}$ such that $j \in F(G')$. This last condition translates to the world of simple CIFs as the requirement that all individuals have smallest winning coalitions of the same cardinality.²³ Our next result is a characterization of simple, anonymous and neutral CIFs.

Theorem 3.2 A CIF $F : \Gamma \rightarrow 2^N$ is simple, anonymous and neutral if and only if F is a symmetric consent rule F^s with $s \in \{1, \dots, \lfloor \frac{n}{2} \rfloor + 1\}$.

Proof. To show the “if” part, take any symmetric consent rule F^s with $s \in \{1, \dots, \lfloor \frac{n}{2} \rfloor + 1\}$. As every symmetric consent rule is, by definition, a generalized consent rule, we know by Theorem 3.1 that F^s is simple and anonymous. It is straightforward to check that symmetric consent rules are neutral. To prove the “only if” part, take any simple, anonymous and neutral CIF F . As F is simple and anonymous, by Theorem 3.1, it is a generalized symmetric consent rule $F^{\bar{s}}$. Consider first the case where n is odd. As $F^{\bar{s}}$ is neutral, hence the smallest winning coalitions of all individuals are of the

²¹This can be expressed, in terms of permutations, as follows: Given any $i, j \in N$, any $\Pi^{ij} : N \rightarrow N$ (as defined in Footnote 18) and any $K \in 2^N$ with $i, j \notin K$, we have $K \in \omega(i) \implies \Pi^{ij}(K) = K \in \omega(j)$.

²²For example, when $N = \{1, 2, 3\}$, the generalized symmetric consent rule $F^{\bar{s}}$ with $s = (1, 1, 2)$ is essentially neutral while individuals 2 and 3 have smallest winning coalitions of different cardinalities.

²³This requirement does not imply essential neutrality, as one can check through the generalized symmetric consent rule $F^{\bar{s}}$ with $s = (2, 2, 2, 3)$ used in the society $N = \{1, 2, 3, 4\}$.

same cardinality, we have $s_i = s_j$ for all $i, j \in N$, showing that F^s is a symmetric consent rule. Now consider the case where n is even. As F^s is neutral, hence the smallest winning coalitions of all individuals are of the same cardinality, we have $s_i \neq s_j$ for some $i, j \in N$ only if $s_i, s_j \in \{\frac{n}{2}, \frac{n}{2} + 1\}$. But essential neutrality implies that there exists no $i, j \in N$ such that $s_i = \frac{n}{2}$ and $s_j = \frac{n}{2} + 1$, showing that $F^{\bar{s}}$ is a symmetric consent rule. ■

Remark that Theorem 3.2 is related to Theorem 2 of Samet and Schmeidler (2003) which characterizes symmetric consent rules in terms of independence, monotonicity, self-duality and Samet-Schmeidler symmetry. It immediately follows from juxtaposing Theorem 2 of Samet and Schmeidler (2003) with our Theorems 2.1 and 3.2 that for simple CIFs, Samet-Schmeidler symmetry is equivalent to the conjunction of anonymity and neutrality.²⁴ On the other hand, on the general domain of CIFs, Samet-Schmeidler symmetry is stronger than anonymity and neutrality.²⁵ So over the domain of simple CIFs, the anonymity and neutrality conditions we propose successfully decompose the symmetry condition of Samet and Schmeidler (2003) which is an incorporation of both equal treatment properties.

4 Characterizing Liberalism

This section contains various characterizations of the liberal CIF. We start by considering a specific profile G^0 with $G_i^0 = \{i\}$ for all $i \in N$, where each individual only considers him/herself as a J . As we remark below, the behavior of simple CIFs on G^0 determines the set of individuals who have the right of self-determination.

Remark 4.1 *Take any simple CIF $F : \Gamma \rightarrow 2^N$ with a family of winning coalitions $\{\omega(i)\}_{i \in N}$. For any $i \in N$, we have $\omega(i) = \{K \in 2^N : i \in K\}$ if and only if $i \in F(G^0)$.*

Liberalism is the assignment to each individual the right of self-determination. So the behavior of CIFs over G^0 is critical in characterizing liberalism. We say

²⁴Recall that although the Samet and Schmeidler (2003) monotonicity condition is stronger than ours, the two monotonicities coincide under independence. See Footnote 6.

²⁵Samet-Schmeidler symmetry implies anonymity directly by the definitions of the two concepts. To see that Samet-Schmeidler symmetry implies neutrality, one can use Π^{ij} permutation for any $i, j \in N$ and for any $G \in \Gamma$. Finally, in a society $N = \{1, 2, 3\}$, the (non-simple) CIF $F : \Gamma \rightarrow 2^N$ which is defined at each $G \in \Gamma$ as $1 \in F(G) \Leftrightarrow \{i \in N : 1 \in G_i\} \supseteq \{2, 3\}$ and for $k \in \{2, 3\}$ we have $k \in F(G) \Leftrightarrow \#\{i \in N : k \in G_i\} \geq 2$ exemplifies a CIF that is anonymous, neutral but not Samet-Schmeidler symmetric.

that a CIF $F : \Gamma \rightarrow 2^N$ satisfies the *weak equal treatment property* (WETP) if and only if $F(G^0) \in \{\emptyset, N\}$. WETP is satisfied *positively* in case $F(G^0) = N$ and negatively when $F(G^0) = \emptyset$. The following theorem characterizes the liberal CIF as the unique simple CIF that satisfies WETP positively.

Theorem 4.1 *A simple CIF $F : \Gamma \rightarrow 2^N$ satisfies the weak equal treatment property positively if and only if F is the liberal CIF.*

Proof. The “if” part immediately follows from the definitions of the liberal CIF and the positive WETP. To see the “only if” part, take any simple CIF F that satisfies WETP positively. By Remark 4.1, we have $\omega(i) = \{K \in 2^N : i \in K\}$ for each $i \in N$, which means that F is the liberal CIF. ■

Theorem 4.1 paves the way to another characterization of the liberal CIF through a liberalism axiom introduced by Kasher and Rubinstein (1997) who say that a CIF $F : \Gamma \rightarrow 2^N$ satisfies the *liberal principle* if and only if for each $G \in \Gamma$, $\exists i \in N$ with $i \in G_i \implies F(G) \neq \emptyset$ and $\exists i \in N$ with $i \notin G_i \implies F(G) \neq N$.

Theorem 4.2 *A simple CIF $F : \Gamma \rightarrow 2^N$ satisfies the weak equal treatment property and the liberal principle if and only if F is the liberal CIF.*

Proof. The “if” part can be seen by checking that the liberal CIF satisfies WETP and the liberal principle. To see the “only if” part, take any simple CIF F that satisfies WETP and the liberal principle. By WETP, we have $F(G^0) \in \{\emptyset, N\}$ while the liberal principle rules $F(G^0) = \emptyset$ out. So $F(G^0) = N$, which means that F satisfies WETP positively and, by Theorem 4.1, F is the liberal CIF. ■

Finally, we show that as the liberal CIF is the unique symmetric consent rule that satisfies liberal principle.

Theorem 4.3 *A simple, neutral and anonymous CIF $F : \Gamma \rightarrow 2^N$ satisfies the liberal principle if and only if F is the liberal CIF.*

Proof. The “if” part can be seen by checking that the liberal CIF satisfies the liberal principle. To see the “only if” part, take any simple, neutral and anonymous CIF F which, by Theorem 3.2, is a symmetric consent rule. Let F satisfy the liberal principle. Note that F satisfies WETP by the definition of a symmetric consent rule. So by Theorem 4.2, F is the liberal CIF. ■

5 Conclusion

We consider a model of aggregating vectors of sets into a set. This mathematical structure is sufficiently rich to allow various interpretations such as the qualification problem where a set of objects is to be partitioned as “goods” and “bads” depending on individuals’ opinions²⁶; electing committees by approval balloting where voters may approve any set of candidates²⁷; deciding over accepting or rejecting a set of issues²⁸ or more generally the aggregation of individual choices into a social choice.²⁹ The interpretation we explore is the collective identity determination problem, proposed by Kasher and Rubinstein (1997), where individuals’ opinions about “Who is a J ” are aggregated into a social decision.³⁰ We introduce the family of simple CIFs which are aggregation rules that can be expressed in terms of winning coalitions. We characterize simple CIFs in terms of three axioms, namely independence, self-duality and monotonicity. Many interesting CIFs of the literature, including (symmetric) consent rules introduced by Samet and Schmeidler (2003), are simple. The class of simple CIFs exhibits the following properties:

- The family of anonymous and neutral CIFs coincides with the family of symmetric consent rules ranging from liberalism to majoritarianism.
- The family of anonymous CIFs coincides with the family of generalized symmetric consent rules.
- All neutral CIFs, but dictatorship, satisfy WETP.
- An anonymous CIF F fails WETP if and only if F is a generalized symmetric consent rule F^s with $s_i = 1$ for some $i \in N$ and $s_j > 1$ for some $j \in N$.

²⁶such as the analysis made by Dimitrov et al. (2004) and Ju (2005a)

²⁷examples of which can be found in Brams et al. (2005b) and Brams et al. (2005a)

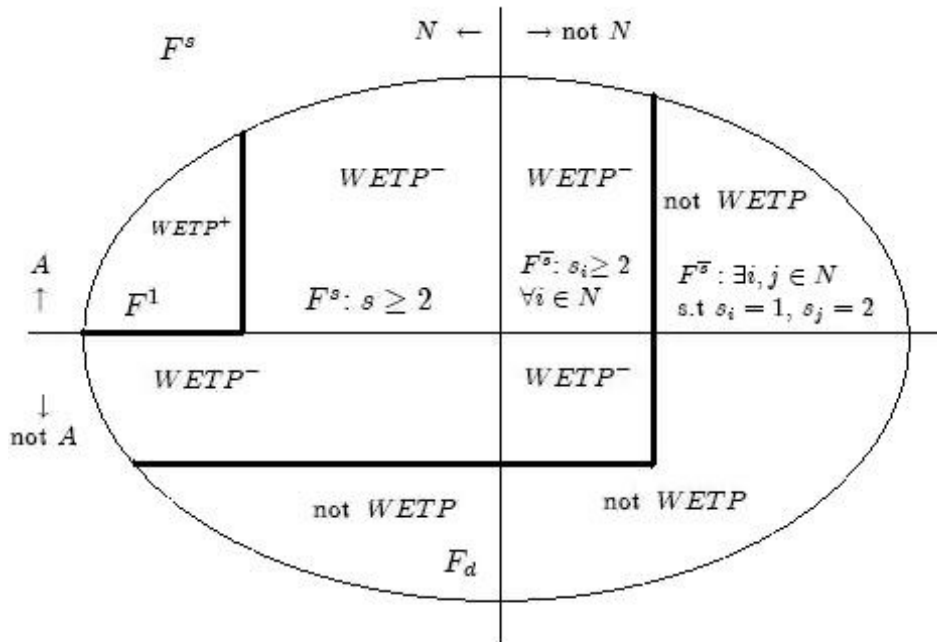
²⁸see Ju (2005b)

²⁹Aleskerov (1999) and Aizerman and Aleskerov (1995) give an excellent treatment of the choice aggregation problem. We wish to say that the plethora of results they establish in an abstract framework can certainly bring further insights to particular applications of aggregating choices, such as the one we consider in this paper. See also Lahiri (2001) and Quesada (2003).

³⁰Under this final interpretation, voters and outcomes coincide, which is not case in the qualification or the committee election problem. This leads to subtleties such as special treatments of “self-qualification” and finer distinctions between the standard anonymity and neutrality conditions of social choice theory - matters which we address and handle in this paper.

- Liberalism is the unique CIF that satisfies WETP positively.
- Among non-anonymous and non-neutral CIFs there are those which do and those which do not satisfy WETP.

The following figure summarizes our findings regarding simple CIFs:



Their ability of expressing the aggregation rule through families of winning coalitions makes simple CIFs of particular interest. Moreover, independence, self-duality and monotonicity are conditions which are very suitable for the identity aggregation context.³¹ It is also to emphasize that among simple CIFs, the liberal one arises as the unique CIF that satisfies positive WETP, which is an intuitive and fairly mild axiom. This supports the idea of endowing individuals with the right of self-determination - hence embracing liberalism as a natural solution to the collective identity determination problem.

³¹though they would not be that appropriate for other interpretations of our model, such as the committee election problem.

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