War with Outsiders Makes Peace Inside*

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Abstract

In many situations there is a potential for conflict both within and between groups. Examples include wars and civil wars and distributional conflict in multitiered organizations like federal states or big companies. This paper models such situations with a logistic technology of conflict. If individuals decide simultaneously and independently about the amount of internal conflict, external conflict and production, there is typically either only internal conflict, or only external conflict - but not both. If each group decides collectively how much each member has to put into the external conflict before the members individually decide on the amounts put into the internal conflict and production, groups choose sufficiently high external conflict in order to avoid internal conflict. This is a model of the "diversionary use of force". We also study the optimal number of groups.

Key Words: conflict, war, rent-seeking, hierarchy, federalism, diversion.

JEL Numbers: D72, D74, H11, H74.

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"The relation of comradeship and peace in the we-group and that of hostility and war towards the other-groups are correlative to each other. The exigencies of war with outsiders are what make peace inside." (Sumner 1906, p. 12).

1 Introduction

Many political and economic situations have a potential for conflict both within and between groups. Military conflicts, wars and civil wars often involve these two layers of conflict.[1] Rent-seeking or appropriative activities in multi-tiered organizations like federal states, firms or universities are other examples.[2]

As William Sumner stated almost 100 years ago, there seems to be a negative correlation between conflicts within a group and conflict between groups.[3] Ralf Dahrendorff even called this a law of the social sciences.[4] This group cohesion effect has been the subject of a large literature in the social sciences. There is support for the group cohesion effect from sociology,

[1] For example, in the Civil War in Spain the two fighting sides or 'groups' consisted of several sub-groups: the old government, the Anarchists, the Communists and others on the left; the Falange with two fractions, the Royalists and the Carlists on the right. On each side, these forces joined to fight against their common enemy; however, there were also severe fights within each of the camps. In the May risings in Barcelona 1937, fighting between Anarchists, Trotskyists, and Communists lead to several hundred deaths. On a smaller scale in April 1937 gun battles took place in Salamanca among the two wings of the Falange. See Beevor (2001), Chapter XIX.

[2] See Wärneryd (1998) on rent-seeking in federal states; Müller and Wärneryd (2001) on conflicts between inside and outside owners of a company; and Inderst, Müller and Wärneryd (2002) on distributional conflict within organizations. See also Konrad (2004). In addition, Glazer (2002) stresses the importance of internal and external rent-seeking in organizations. See also Garfinkel (1994) for a study of interrelations between domestic politics and international conflict.

[3] Similar points were observed much earlier by chroniclers of the Italian city republics, as Waley (1988, p. 117) reports: "Even in the eleventh century Milanese chroniclers had remarked of their fellow citizens that 'when they lack external adversaries they turn their hatred against each other'. This well-founded observation was to become a commonplace for other cities too. Florence was built under the signs of Aries and Mars, says Malispini: 'our ancestors were always fighting battles and wars and when they had no other opponent they fought among themselves'".

[4] "It appears to be a general law that human groups react to external pressure by increased internal coherence." Dahrendorf (1964), p. 58, cited after Levy (1989), p. 261.

psychology, and anthropology.[5] A recent case in point was noted by The Economist: George W. Bush had "a year of stratospheric popularity after the attacks of September 11th 2001 and a shorter gust of support after the war began in Iraq"[6].

This paper is one of the first studies of an economic model of conflict for such situations with simultaneous conflicts at different levels. Economic models of conflict start with the fact that property rights are not always completely enforced. Thus individuals can, and do, engage in activities that are not productive - they do not increase the size of the pie but instead are aimed at increasing one's share of the pie.[7] Each player decides how to allocate his resources to three kinds of activities: to production, to rent-seeking (or fighting) for one's own group, and to rent-seeking (or fighting) within one's own group. We assume that the technology of conflict is given by a logistic or difference form contest success function. The logistic technology of conflict has been used frequently in the literature. Skaperdas (1996) gives an axiomatic foundation. Hirshleifer (1991) argues that the logistic contest success function is in line with a number of stylized facts of warfare. We show that in our model there is an extreme form of the group cohesion effect: in equilibrium there is conflict either between or within groups, but - except for a nongeneric case - not both at the same time. This result is due to the property of difference form contest success functions that, in a symmetric situation where all players choose the same rent-seeking efforts, the marginal returns to rent-seeking are independent of the total amount of rent-seeking. Thus, if marginal returns to intra-group rent-seeking are higher than those to inter-group rent-seeking, there is only internal conflict, and vice versa. Changes in the environment can switch the equilibrium from conflict between groups and peace within groups to conflict only within groups and peace between groups, and vice versa.

In the basic model, we assume that all players decide simultaneously and

[5] For example, Stein concludes in his excellent survey of the literature up to the mid 1970s that, under some conditions, "external conflict does increase internal cohesion" (1976, p. 165). Dion (1979) and Fisher (1990) come to similar conclusions in their surveys of the literature. See Bornstein (2003) for a survey of related experimental work.

[6] "On the back foot", June 23rd 2005.

[7] Such activities have been called 'appropriation' (Grossman 1994), 'rent-seeking' (Tullock 1980), 'power seeking' (Rajan and Zingales 2000), 'influence activities' (Milgrom 1988, Milgrom and Roberts 1990), 'coercive activities' (Skaperdas 1992) and simply 'stealing' or 'fighting' (Hirshleifer 1988, and his essays collected in Hirshleifer 2001) in the literature.

independently how to allocate their resources to the three kinds of activity. This is a realistic description in situations with somewhat amorphous conflicts, like rent-seeking in organizations, or chaotic situations within civil wars. However, in many applications, the decision about external conflicts has a different character than that over internal conflicts. For example, decisions about inter-state wars are usually taken by some political process, e.g. by a leader, or by an elected government or parliament and not by the subjects individually. We model this in an extension to the basic model where we assume that each group first decides how many resources each group member has to put in the external conflict before the individuals decide about the allocation of their remaining resources to internal fighting and production. The outcome is quite different: groups commit to levels of external conflict that are sufficiently high to prevent any internal conflict occurring.

Thus, we propose a model for the old idea that external conflict is used to prevent internal conflict. This "diversionary theory of war" dates back to Jean Bodin (1556) ("the best way of preserving a state, and guaranteeing it against sedition, rebellion, and civil war is to keep the subjects in amity one with another, and to this end, to find an enemy against whom they can make common cause" (Six Books of the Commonwealth, translated by M. J. Tooley 1955, Book V Chap. V)). Jonathan Swift (1720) ("Wise princes find it necessary to have wars abroad to keep peace at home"[8]) and William Shakespeare (1591) ("Be it thy course to busy giddy minds / With foreign quarrels", Henry IV Part II, Act 4 Scene 5) expressed similar ideas. The idea can be found in many historical writings and in case studies of specific wars (see e.g. Joll (1999) on Germany and others in World War I , and Levy and Vakili (1992) on Argentina in the Falklands/Malvinas War)[9].

Two important contributions are those of Hess and Orphanides (1995, 2001) who explain the occurrence of diversionary wars by the voters' incomplete information about the abilities of a leader.[10] A leader with a reelection

[8] The quote is from The History of Martin, A digression on the nature usefulness & necessity of wars and quarels. See Swift (1720 [1958]), p. 305. There are some doubts whether Swift is indeed the author.

[9] A list of classic cases is given by Levy and Vakili (1992), Footnote 5 on p. 138. In the quantitative empirical work on the issue, however, findings are mixed and no consensus has emerged. See Levy (1989) and Levy (1998) for surveys; recent contributions include Chiozza and Goemans (2003), Mc Laughlin Mitchell and Prins (2004), and Fordham (2005).

[10] See also Smith (1996).

motive and a bad internal performance (say, high unemployment or inflation) may wish to engage in external war in order to convince the voters of his high military abilities. Contrary to Hess and Orphanides, we assume complete information and show that there is nevertheless a possibility of diversionary wars.

Our model also contributes to the explanation of the emergence of several internal wars after the demise of the cold war. The end of the superpower confrontation diminished incentives to engage in external conflict, because of less pronounced ideological differences between the two blocks and better collaboration in international organizations. It also lowered the ability of the former Eastern Block members to commit themselves to a level of external conflict sufficient to eliminate internal conflicts. As one would expect from our model, internal conflicts flared up; the wars in the Balkans, in Chechnya, Georgia, and Moldova are cases in point.

We also derive normative implications for the optimal design of an organization that is ridden by potential conflict both between and within its groups. For a given size of the organization, how many groups should there be? With only one group, there is obviously only intra-group conflict. Increasing the number of groups decreases the amount of conflict, until at some point conflict within groups ceases and conflicts between the groups start. After that point, a further increase in the number of groups leads to more conflict and to less production in equilibrium. We show that, if the technology of conflict is as decisive in the inter-group conflict as it is in the intra-group conflict, then the organization should have as many groups as there are individuals in each group. If the inter-group conflict is more decisive, the optimal number of groups is smaller; and vice versa: if the intra-group conflict is more decisive, there should be more groups.

The paper adds to the literature on the trade-off between production and appropriation. Important contributions are Skaperdas (1992), Grossman (1991, 1994), Grossman and Kim (1995) and Hirshleifer (1988, 2001); an important recent contribution is Baker (2003). See Hirshleifer (1995) and Skaperdas (2003) for surveys. In a companion paper, one of us has studied simultaneous inter- and intra-group conflict with a ratio form conflict technology (Münster 2005). However, Münster (2005) does not look at the case where groups commit to external conflict before individuals decide on production and internal conflict, and hence does not study the diversionary use of force. Moreover, with the ratio form technology, equilibria are always characterized by simultaneous conflicts both between and within groups. In addition, the normative implications differ. Most importantly, with the ratio type of technology and equally decisive contests between and within groups, the number of groups has no influence on total conflict and production. This is in stark contrast to the findings for the logistic or difference form contest technology discussed in this paper.

Our paper is also related to the literature on collective rent-seeking. Early contributions (Nitzan 1991) assumed that the distribution within a group is given by some sharing rule which might depend on the inter-group rent-seeking efforts, but did not model the internal conflict explicitly. Katz and Tokatlidu (1996) study a two stage rent-seeking game where in the first stage, individuals decide how much to invest in inter-group rent-seeking, and in the second stage there is internal rent-seeking. Wärneryd (1998) applies a similar model to rent-seeking in federal states, arguing that a federal state may actually lead to less rent-seeking than a centralized state, although it adds one additional layer of conflict. Müller and Wärneryd (2001) have a related result on inside versus outside ownership; and Inderst, Müller, and Wärneryd (2002) apply this logic to the allocation of scarce resources within a firm. Konrad and Leininger (2005) present a model where a group can resolve its collective action problem in the supply of inter-group rent-seeking effort because the ensuing intra-group rent-seeking contest has multiple equilibria. In contrast to our paper, none of these papers studies simultaneous inter- and intra-group conflicts, or the case where groups collectively decide about how much every member must put into the inter-group conflict. In addition, in these rent-seeking models, the size of the contested rent is usually assumed to be exogenous, whereas our model is concerned with the trade-off between production and rent-seeking.

Several papers study the effects of different degrees of external conflict. Alesina and Spolaore (2003, 2005) study the relation between the size of countries and the incidence of international war. They argue that the equilibrium size of a country depends on the importance of international war. Better defined international property rights may lead to smaller countries and hence a higher number of countries. This has the effect that the number of international conflicts may actually increase. Alesina and Spolaore (2005) focus on international wars and abstract from internal conflicts once a country is formed. Our analysis complements theirs, since we focus explicitly on the occurrence of internal as well as international conflicts. On the other hand, we abstract from issues of country formation of secession which are at the center of their analysis. Garfinkel (2004) looks at how an increased external risk influences into internal conflict. However, she does not explicitly model the behavior of the competing groups in an external conflict.

Section 2 presents the basic model of simultaneous inter- and intra-group conflicts, section 3 studies its equilibria. Section 4 deals with the case where the individuals collectively decide about the amount of inter-group conflict before engaging in production and internal conflict. Here we expose the diversionary use of force argument in our model. Section 5 deals with the normative issues of the optimal number and size of groups. Section 6 concludes. The appendix collects some of the longer proofs.

2 The model

There are n identical individuals who are divided into G groups of equal size $m = n/G$. Each individual is endowed with one unit of time. In the basic model, an individual i in group g has three choice variables: productive effort e_{ig}, intra-group rent-seeking effort x_{ig} and inter-group rent-seeking effort y_{ig}. The individuals simultaneously and independently decide how to allocate their resources to these three activities.[11] The budget constraints of the individuals are

$$e_{ig} + x_{ig} + y_{ig} = 1.$$

Output is given by the production function

$$q = f\left(\sum_{g=1}^{G}\sum_{i=1}^{m} e_{ig}\right)$$

where $f(0) = 0$[12] and $f'(z) > 0$ for all $z \geq 0$. In addition, we assume that f is log-concave. Although we focus the discussion below on the opportunity costs of fighting, this production function also captures potential other negative effects, for example when fighting destroys output, as long as the amount of destruction depends on the sum of fighting. To see this, consider the

[11] For some applications it is more appropriate to assume that the amount of external conflict is chosen collectively, by some political process, before the individuals engage in production and internal conflicts. We model this in section 4.

[12] Often fighting is not only over today's output, but also over (say) land, natural resources, or valuable items produced in the past. We could capture this by assuming $f(0) > 0$. In this case, there are additional corner solutions where there is only fighting, but the main results below carry over.

following production function

$$F\left(\sum_{g=1}^{G}\sum_{i=1}^{m} e_{ig}, \sum_{g=1}^{G}\sum_{i=1}^{m}(x_{ig}+y_{ig})\right)$$

where F is increasing in its first argument and decreasing in its second. Using the budget constraints we can write

$$F\left(\sum_{g=1}^{G}\sum_{i=1}^{m} e_{ig}, n-\sum_{g=1}^{G}\sum_{i=1}^{m} e_{ig}\right) = f\left(\sum_{g=1}^{G}\sum_{i=1}^{m} e_{ig}\right).$$

The distribution of output depends on the rent-seeking activities. The share that goes to group g is denoted by p_g which is a function of all the inter-group rent-seeking activities. Thus the amount that group g gets is $p_g q$. From that amount, individual i in group g gets the share r_{ig} which depends on the intra-group rent-seeking activities of the members of group g. The payoff of individual i in group g thus is

$$u_{ig} = r_{ig} p_g q.$$

We assume a logistic technology of conflict. Individual i in group g gets the fraction

$$r_{ig} = \frac{\exp(a x_{ig})}{\sum_{j=1}^{m}\exp(a x_{jg})}.$$

The parameter $a > 0$ describes the decisiveness of the intra-group contest. If a is small, the rent-seeking efforts will have little influence on the division of output, whereas if $a \to \infty$ then small differences in rent-seeking efforts are decisive. We think of a being determined by both technological factors (for example police technology) and institutional factors pertaining to the security of property within a group.

Similarly, group g gets the fraction

$$p_g = \frac{\exp(b\sum_{j=1}^{m} y_{jg})}{\sum_{k=1}^{G}\exp(b\sum_{j=1}^{m} y_{jk})}.$$

The parameter $b > 0$ describes the decisiveness of the inter-group contest. Like a, this is determined by technological and institutional factors. However,

b may differ from a since both the technology and the relevant institutions differ in contests between groups from those in contests within groups. For example, a higher importance of international institutions and methods of peaceful international conflict resolution would mean that there is a lower incentive to engage in international fighting, which can be captured by a lower b in our model. Similarly, the introduction of new means in inter-group conflicts (as the use of airplanes in the terrorist attacks of September 11, 2001) can be understood as a higher decisiveness of inter-group conflict.

The logistic contest success function is continuous, even when all contestants choose zero rent-seeking. This leads to a possibility of corner solutions where some individuals choose zero rent-seeking effort. Moreover, adding a constant to all the rent-seeking efforts does not change the shares of the contestants. The shares thus depend only on the difference of the rent-seeking efforts; the logistic contest success function is therefore also known as the difference form technology of conflict.[13]

Since the game is completely symmetric, we will focus on symmetric equilibria. The game also has asymmetric equilibria, but these have similar properties.

3 What type of conflict?

Use the budget constraint to eliminate e_{ig} from the objective function and write

$$u_{ig} = r_{ig} p_g f\left(\sum_{k=1}^{G}\sum_{j=1}^{m}(1 - x_{jk} - y_{jk})\right). \tag{1}$$

Individual i in group g solves the following maximization problem

$$\max_{x_{ig},\, y_{ig}} u_{ig} \text{ subject to}$$
$$x_{ig} \geq 0,\ y_{ig} \geq 0,\ x_{ig} + y_{ig} \leq 1.$$

With the logistic conflict technology, an individual's share of total output

[13] This is the main property of the logistic contest success function that we will use below; it corresponds to Axiom A7 in the axiomatization of contest success functions by Skaperdas (1996). We could allow for more general difference form contest success functions, dropping Axioms A4 and A5 in Skaperdas (1996), without affecting our main results.

is always strictly positive: $r_{ig}p_g > 0$. Thus, each individual can guarantee himself a strictly positive utility by choosing strictly positive productive effort; hence in any equilibrium, output is strictly positive. Therefore, in a symmetric equilibrium, the constraint $x_{ig} + y_{ig} \leq 1$ is not binding.

The objective function is concavifiable (we show in appendix A1 that $\ln u_{ig}$ is strictly concave in (x_{ig}, y_{ig})). Moreover, the constraint set is convex. Therefore, the following first order conditions are both necessary and sufficient for a maximum:

$$\frac{\partial u_{ig}}{\partial x_{ig}} \leq 0, \; x_{ig} \geq 0, \; x_{ig}\frac{\partial u_{ig}}{\partial x_{ig}} = 0, \tag{2}$$

$$\frac{\partial u_{ig}}{\partial y_{ig}} \leq 0, \; y_{ig} \geq 0, \; y_{ig}\frac{\partial u_{ig}}{\partial y_{ig}} = 0. \tag{3}$$

The first inequality in (2) can be expressed as

$$a\frac{\sum_{j\neq i}^{m} \exp(ax_{jg})}{\sum_{j=1}^{m} \exp(ax_{jg})} \leq \phi\left(\sum_{k=1}^{G}\sum_{j=1}^{m}(1 - x_{jk} - y_{jk})\right),$$

where $\phi(z) := f'(z)/f(z)$. Note that, by our assumption that f is log-concave, $\phi'(z) < 0$ and an inverse function ϕ^{-1} exists. Focussing on a symmetric situation where $x_{jk} = x$ and $y_{jk} = y$ for all individuals j and groups k, we get

$$a\frac{m-1}{m} \leq \phi\left(n\left(1 - x - y\right)\right). \tag{4}$$

Inequality (4) describes the trade-off between production and intra-group rent-seeking in a symmetric equilibrium. If it holds with strict inequality, then x must be zero: the marginal benefit of intra-group rent-seeking is smaller than the opportunity cost of foregone production.

The first inequality in (3) gives us

$$b\frac{\sum_{k\neq g}^{G} \exp(b\sum_{j=1}^{m} y_{jk})}{\sum_{k=1}^{G} \exp(b\sum_{j=1}^{m} y_{jk})} \leq \phi\left(\sum_{k=1}^{G}\sum_{j=1}^{m}\left(1 - x_{jk} - y_{jk}\right)\right).$$

Imposing symmetry, we get

$$b\frac{G-1}{G} \leq \phi\left(n\left(1 - x - y\right)\right). \tag{5}$$

Inequality (5) describes the trade-off between production and inter-group rent-seeking in a symmetric equilibrium. If it holds with strict inequality, then y must be zero, because the marginal benefit of inter-group rent-seeking is smaller than the opportunity cost of foregone production.

There are four cases to consider. These are given in definition 1 below and illustrated in figure 1.

Definition 1 *Let case NO be defined by*

$$\phi(n) \geq \max\left\{b\frac{G-1}{G}, a\frac{m-1}{m}\right\}, \tag{NO}$$

case INTRA by

$$a\frac{m-1}{m} > \max\left\{b\frac{G-1}{G}, \phi(n)\right\}, \tag{INTRA}$$

case INTER by

$$b\frac{G-1}{G} > \max\left\{a\frac{m-1}{m}, \phi(n)\right\}, \tag{INTER}$$

and finally case SPECIAL by

$$b\frac{G-1}{G} = a\frac{m-1}{m} > \phi(n). \tag{SPECIAL}$$

Which of these cases we are in, depends on the decisiveness of internal and external conflicts, on the number and size of the groups, and on the production technology. We begin with case NO, where the opportunity cost of rent-seeking in terms of forgone production are prohibitively high.

Lemma 1 *In case NO, there is a unique symmetric equilibrium where all effort is put into production:*

$$e = 1 \text{ and } x = y = 0.$$

Proof. To see that this is an equilibrium, note that the first order conditions hold at $(x, y) = (0, 0)$. To show uniqueness, suppose, to the contrary, that $(x', y') \neq (0, 0)$ is an equilibrium. Since $\phi(n(1-x-y))$ in increasing in x and in y, it follows from condition (NO) that both (4) and (5) must hold with strict inequality. This implies that $x' = y' = 0$, a contradiction. ■

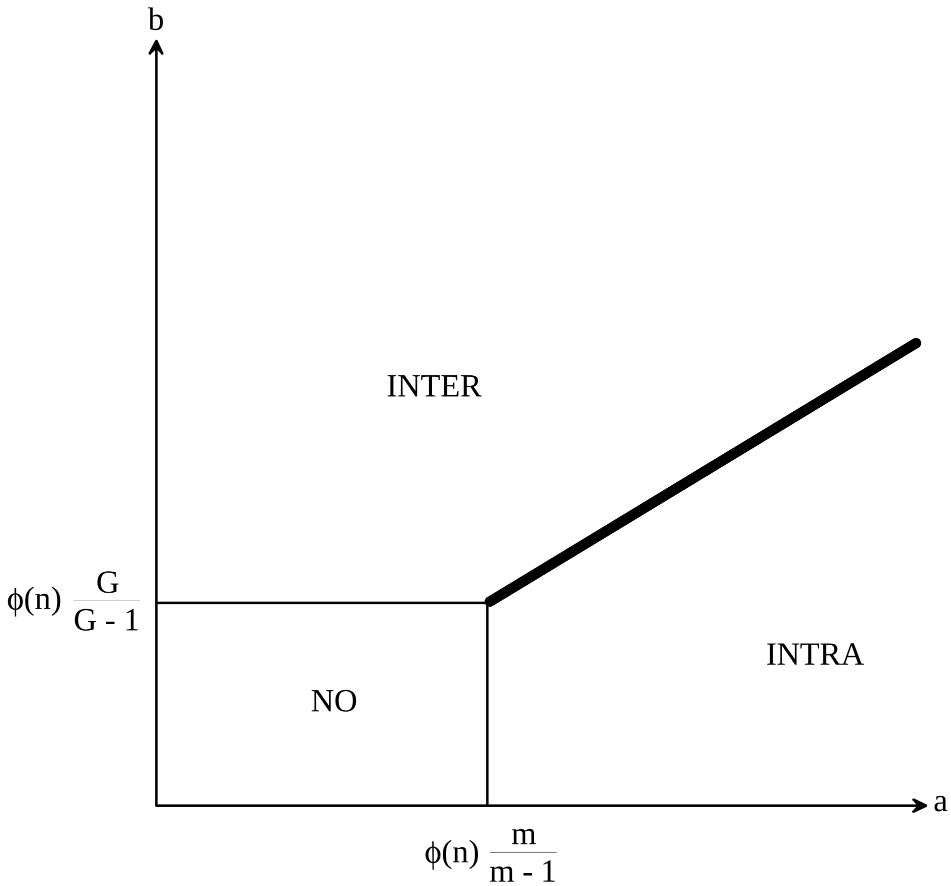

Figure 1: Here we illustrate the cases given in definition 1. The bold line corresponds to case SPECIAL.

Lemma 1 describes case NO where property is relatively secure and thus the parameters a and b are relatively small. In this case, the individuals do not engage in rent-seeking activities. However, if inequality (NO) does not hold, there will be rent-seeking in equilibrium. Typically, there is conflict only within groups, or only between groups - but not both. For example, if a is big relative to b, then there will be rent-seeking within the groups, and no rent-seeking between groups; this is case INTRA dealt with in lemma 2 below. On the other hand, if b is big relative to a, then there is only inter-group rent-seeking; this is case INTER dealt with in lemma 3.

Lemma 2 *In case INTRA, there is a unique symmetric equilibrium where there is no rent-seeking between groups ($y = 0$). Intra-group rent-seeking $x \in (0, 1)$ is given by*

$$\frac{a\,(m-1)}{m} = \phi\,(n\,(1-x))\,, \tag{6}$$

and productive effort is $e = 1 - x$.

Proof. First note that (6) defines a unique x. Second, $x > 0$ by (INTRA). Third, $x < 1$ since $\lim_{z\to 0} \phi\,(z) = \infty$. Thus no constraints are violated. Existence follows from the fact that the first order conditions hold. For uniqueness, we proceed in four steps.

1. As argued above, in any equilibrium $e > 0$ and hence $x + y < 1$.
2. $x = y = 0$ is not an equilibrium, since then (4) is violated.
3. Suppose that $y > 0$ in equilibrium. This implies that (5) holds with equality. Together with (INTRA), this contradicts (4).
4. It follows from steps 1 to 3 that in equilibrium we have $0 = y < x < 1$. Hence (4) holds with equality. This implies (6).

■

Lemma 3 *In case INTER, there is a unique symmetric equilibrium where there is no intra-group rent-seeking ($x = 0$). Inter-group rent-seeking $y \in (0, 1)$ is determined by*

$$b\frac{G-1}{G} = \phi\,(n\,(1-y))\,, \tag{7}$$

and productive effort is $e = 1 - y$.

Proof. Similar to the proof of lemma 2. ■

Finally, we have to consider case SPECIAL. Although this is a nongeneric case (it corresponds to the bold line in figure 1), it will turn out to be interesting from a normative point of view in section 5. In case SPECIAL, (4) and (5) are no longer independent. This leads to a multiplicity of symmetric equilibria.

Lemma 4 *In case SPECIAL, there exists a continuum of symmetric equilibria where $x + y$ is determined by*

$$a\frac{m-1}{m} \equiv b\frac{G-1}{G} = \phi\left(n\left(1-x-y\right)\right), \tag{8}$$

and $e = 1 - x - y \in (0, 1)$.

Proof. The first order conditions hold at all e, x and y satisfying (8). ■

Note that all the equilibria described in lemma 4 have the same amount of production. The following proposition sums up the discussion and gives some comparative statics.

Proposition 1 *i) Only in the nongeneric case SPECIAL there is conflict both within and between groups. Typically, if there is conflict at all, it takes place either within groups, or between groups, but not both.*
ii) Conflict within groups is more likely if conflicts within groups are very decisive (a is large) and the number of groups G is small, holding the n constant. When conflict within groups occurs, then its intensity is increasing in a and decreasing in G.
iii) Conflict between groups is more likely if conflicts between groups are very decisive (b is large) and G is large, holding the n constant. When conflict between groups occurs, its intensity is increasing in b and in G.

Proof. Part i) is clear from lemmas 1-4. For part ii), note that $(m-1)/m \equiv (n-G)/n$ since $n \equiv mG$. Thus, holding n constant, $(m-1)/m$ decreases in G. The result follows from $\phi'(z) < 0$ and lemma 2. Similarly, part iii) follows from lemma 3. ■

In the model we have an extreme form of the group cohesion effect: there is either conflict within or between groups. In particular, a change in the technology of conflict can change the situation from external conflict with internal peace to internal conflict with external peace. This may contribute to an explanation of several civil wars that occurred after the end of the cold war. Diminished ideological differences, and better possibilities for peaceful cooperation in international organizations all meant that the benefits from engaging in external conflict between the former two blocks were lowered. In terms of the model, b went down. Accordingly, the equilibrium switched to peace between the two former blocks, but then with internal conflicts.

4 The diversionary use of force

As we argued in the introduction, decisions about external conflict are often taken by some political process which is different from the simultaneous and independent optimization studied above in the basic version of our model. Therefore, in this section we study what happens if the amount of external conflict is chosen collectively, by some political process, before the individuals engage in production and internal conflicts. To be more precise, we propose the following two stage game:

1. Each group decides about the amount y_g that each group member has to put into the external conflict.

2. After observing all the decisions taken in stage 1, individuals simultaneously and independently decide how to allocate their remaining resources to production and internal fighting. That is, individual i in group g chooses x_{ig} and e_{ig} such that $x_{ig} + e_{ig} = 1 - y_g$.

There are several ways to think of the political decision process in stage 1. In the model these all lead to the same result. One of the individuals in a group might be a dictator who chooses y_g in order to maximize his utility. Or each group might delegate the decision right to one of its members, a "politician" who has to decide about the amount of resources to put into external conflict. Finally, a group might just vote: due to the assumption of identical individuals, the decision is unanimous.

We solve the game by backward induction. Section 4.1 studies the equilibrium of the subgames in stage 2 for all possible choices taken in stage 1. In section 4.2 we solve for the subgame perfect equilibrium.

4.1 The second stage: internal fighting

In the second stage individuals choose the effort they put into intra-group rent-seeking and production. In their choices they are constrained by the amount of resources that has already been devoted to the external conflict in the first stage. Denote the objective function of individual i in group g on the second stage by v_{ig}; and let x_{-ig} denote the vector of all the x_{jk} except x_{ig}. In stage 2, individual i in group g maximizes the following over x_{ig}

$$v_{ig}\left(x_{ig}, x_{-ig}; y_1, ..., y_G\right) = p_g \frac{\exp\left(a x_{ig}\right)}{\sum_{j=1}^{m} \exp\left(a x_{jg}\right)} f\left(\sum_{k=1}^{G} \sum_{j=1}^{m} \left(1 - y_k - x_{jk}\right)\right)$$

subject to

$$0 \leq x_{ig} \leq 1 - y_g.$$

This is a concavifiable objective function with a convex constraint set. In a solution, one of the following first order conditions has to hold:

$$\frac{\partial v_{ig}}{\partial x_{ig}} \leq 0, x_{ig} = 0 \quad \text{or} \tag{9}$$

$$\frac{\partial v_{ig}}{\partial x_{ig}} = 0, x_{ig} \in \left(0, 1 - y_g\right) \quad \text{or} \tag{10}$$

$$\frac{\partial v_{ig}}{\partial x_{ig}} \geq 0, x_{ig} = 1 - y_g. \tag{11}$$

Moreover, if one of these conditions holds, this is sufficient for a maximum. These optimality conditions are similar to line (2) above, with the additional constraint that x_{ig} cannot be bigger than $1 - y_g$. Since groups may have chosen different amounts of external fighting in the first stage, we cannot assume a symmetric solution where all the x_{ig}s are equal. This also means that for some individuals the constraint that internal fighting cannot exceed $1 - y_g$ might be binding; line (11) takes this into account.

Calculating the derivative, we find that

$$\frac{\partial v_{ig}}{\partial x_{ig}} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \Leftrightarrow \frac{a \sum_{j \neq i} \exp\left(a x_{jg}\right)}{\sum_{j=1}^{m} \exp\left(a x_{jg}\right)} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \phi \left(n - m \sum_{k=1}^{G} y_k - \sum_{k=1}^{G} \sum_{j=1}^{m} x_{jk}\right) \tag{12}$$

From line (12) and the first order conditions (9)-(11), one can show the following lemma.

Lemma 5 *Any equilibrium of the second stage is within-group symmetric: all players $i = 1, ..., m$ in a given group k choose the same $x_{ik} = x_k$.*

Proof. See appendix A2. ■

To see the logic behind this, consider the case where players 1 and 2 in a group g choose to engage in both internal conflict and production. Then the first order conditions imply that

$$\frac{\frac{\partial r_{1g}}{\partial x_{1g}}}{r_{1g}} = \phi\left(n - m\sum_{k=1}^{G} y_k - \sum_{k=1}^{G}\sum_{j=1}^{m} x_{jk}\right) = \frac{\frac{\partial r_{2g}}{\partial x_{2g}}}{r_{2g}}$$

and this, in turn, implies that $x_{1g} = x_{2g}$. Of course, we have to check for corner solutions as well; but here basically the same logic goes through.

Using within-group symmetry, line (12) simplifies to

$$\frac{\partial v_{ig}}{\partial x_{ig}} \left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} 0 \Leftrightarrow \frac{a\,(m-1)}{m} \left\{\begin{array}{c} > \\ = \\ < \end{array}\right\} \phi\left(n - m\sum_{k=1}^{G}(y_k + x_k)\right) \tag{13}$$

As one can imagine from our analysis in the last section, there are two cases to consider. There is a threshold which depends on the choices in the first stage, such that if a is below the threshold, there is no internal fighting and only production in stage 2. Lemma 6 below makes this precise. On the other hand, if a is bigger than the threshold, then there is internal conflict (see lemma 7 below).

Lemma 6 *If*

$$\phi\left(n - m\sum_{k=1}^{G} y_k\right) \geq \frac{a\,(m-1)}{m}, \tag{14}$$

then there exists a unique equilibrium of the subgames in stage 2 with $x_{ig} = 0$ for all individuals i and groups g. Payoffs are

$$u_{ig} = \frac{\exp\left(bmy_g\right)}{\sum_{k=1}^{G}\exp\left(bmy_k\right)}\frac{1}{m}f\left(n - m\sum_{k=1}^{G} y_k\right). \tag{15}$$

Proof. See appendix A3. ■

To gain some intuition, observe that (14) is likely to hold if there has been a lot of external fighting in stage 1. Hence there are few resources left over in stage 2. Even if all the remaining resources are allocated to production, the marginal incentives to produce are higher than the marginal incentives to engage in internal conflict. Thus, there is no internal fighting.

Lemma 7 *If inequality (14) does not hold, then in any equilibrium of the subgames in stage 2 there is internal fighting. The total amount of internal fighting is determined by*

$$\frac{a\,(m-1)}{m} = \phi\left(n - m\sum_{k=1}^{G}(y_k + x_k)\right). \tag{16}$$

Payoffs in all equilibria are

$$u_{ig} = \frac{\exp\left(bmy_g\right)}{\sum_{k=1}^{G}\exp\left(bmy_k\right)}\frac{1}{m}f\left(\phi^{-1}\left(\frac{a\,(m-1)}{m}\right)\right). \tag{17}$$

Proof. See appendix A4. ■

Lemma 7 describes the case where there was relatively little fighting in the first stage. Here, if all the remaining resources were allocated to production, the marginal incentives to fight internally would be higher than the marginal incentives to produce. Thus, there is some internal fighting going on, the amount being determined by the trade-off between internal fighting and production. Since expression (16) only determines the total amount of internal fighting, the equilibrium will typically be not unique. All equilibria, however, are payoff equivalent.

An interesting feature of the equilibrium utilities of the subgames in the second stage is that, in the case with internal fighting (lemma 6), equilibrium production does not depend on the choices made in the first stage. If there are more resources left over from the first stage, this simply leads to more internal conflict in the second stage. Conversely, increasing y_g gives groups g a higher share p_g of output, without decreasing production. Of course, this is no longer true once there is so much external fighting in stage 1 that (14) holds.

4.2 The first stage: external fighting

To analyze the equilibrium choices of the first stage, we again have to consider the cases distinguished in definition 1. In case NO, where both a and b are relatively small, there will be no conflict at all, and $y_g = 0$ for all groups. Moreover, in case INTER where b is relatively big, the equilibrium amount of inter-group conflict will be determined by a trade off between inter-group rent-seeking and production, exactly as in lemma 3 above. Given this amount of inter-group fighting, there is no internal conflict in stage 2.

The case INTRA where a is relatively big is different, however. In this case, the equilibrium amount of inter-group conflict is determined by the condition that there is no internal conflict in the second stage. That is, $\sum y_k$ is determined such that (14) holds with equality. Again we focus on symmetric equilibria where $y_1 = ... = y_G =: y$. Proposition 2 sums up the results.

Proposition 2 *Suppose groups commit to the amount of external fighting that each individual has to put into the inter-group conflict, before individuals decide on the allocation of their remaining resources to internal conflict and production. There is a symmetric subgame perfect equilibrium, where the equilibrium choices of external fighting are as follows.*
In case NO, $y_1 = ... = y_G = 0$.
In case INTRA, y *is given by*

$$\frac{a\,(m-1)}{m} = \phi\,(n\,(1-y))\,. \tag{18}$$

In the cases INTER and SPECIAL, y *is given by equation (7) in lemma 3.*
In any case, there is no internal fighting on the equilibrium path.

Proof. See appendix A5. ■

The basic insight behind proposition 2 is as follows. In the first stage, the groups will spend enough on the external conflict in order to ensure that there is no internal fighting. This can be seen directly from equation (17): as long as $\sum y_k$ is small enough such that we are in the case with internal fighting described in lemma 6, utility of a group g is strictly increasing in y_g. Thus, on the equilibrium path there is no internal fighting. Rather, groups use the external conflict in order to get rid of their internal quarrels. In case

INTRA, the groups choose their inter-group fighting such that there is no internal conflict. Quite literally, war with outsiders makes peace inside!

An interesting implication is that in case INTRA, how much *external* fighting occurs depends on the technology of *internal* conflict:

Corollary 1 *In case INTRA, an increase in internal insecurity a leads to more external fighting y, while changes in external insecurity b do not affect the amount of external fighting.*

Proof. From (18). ∎

Notice that in case INTRA, groups start fighting against each other even if b is very small. In the extreme case where $b = 0$, external fighting has no influence on the share of a group and thus groups do not have an appropriation motive for external fighting. Nevertheless, even if $b = 0$ there is a subgame perfect equilibrium where groups fight enough to prevent internal conflict. There are additional equilibria with internal fighting as well in this case; they disappear as soon as b is strictly positive, however small. This is thus a model of the "diversionary use of force".

5 In optimum both types of conflict

This section turns to the normative implications of the model. In particular, how many groups should an organization have, if it is ridden with potential conflict both within and between its groups? It turns out that the answer to this question does not depend on whether groups can commit to their external fighting before deciding on internal fighting and production. As the following corollary shows, equilibrium utility is identical in both versions of the model.

Corollary 2 *In case NO equilibrium utility is $u_{ig} = f(n)/n$. In case INTRA equilibrium utility equals*

$$u_{ig} = \frac{1}{n} f\left(\phi^{-1}\left(\frac{a(m-1)}{m}\right)\right).$$

In the cases INTER and SPECIAL, equilibrium utility equals

$$u_{ig} = \frac{1}{n} f\left(\phi^{-1}\left(\frac{b(G-1)}{G}\right)\right).$$

Proof. Case NO: follows directly from lemma 1 and proposition 2 (case NO), respectively.

Case INTRA: from lemma 2 and proposition 2 (case INTRA), respectively, it follows that total productive effort is

$$\sum_{g=1}^{G}\sum_{i=1}^{m} e_{ig} = \phi^{-1}\left(\frac{a\,(m-1)}{m}\right).$$

Inserting this in the utility function gives the result.

Case INTER: Similarly from lemmas 3 and 4, and proposition 2 (case INTER). ■

As long as (INTRA) holds, productive effort is increasing in the number of groups holding constant the size n of the organization.[14] Hence equilibrium utility is increasing. But after some point, we switch to case INTER, and here productive effort and equilibrium utility are decreasing in the number of groups. Thus, in order to minimize conflict, the organization should be designed such that we are in case SPECIAL, where there are simultaneous inter- and intra-group conflicts. In this sense, it is optimal to have both types of conflict. Proposition 3 makes this precise.

Proposition 3 *For given n, an optimal number of groups is given by (ignoring integer problems)*

$$G^{*} = \frac{1}{2a}\left((a-b)\,n + \sqrt{n\left(4ab + n\,(b-a)^{2}\right)}\right). \tag{19}$$

Proof. If $a\,(m-1)\,/m \equiv a\,(n-G)\,/n > b\,(G-1)\,/G$, equilibrium utility is weakly increasing in G by Corollary 2 and the fact that ϕ^{-1} is decreasing. On the other hand, if $a\,(n-G)\,/n < b\,(G-1)\,/G$, equilibrium utility is weakly decreasing in G. Thus an optimal G solves

$$a\frac{n-G}{n} = b\frac{G-1}{G}. \tag{20}$$

[14] Recall that since $n \equiv mG$, we have $(m-1)\,/m \equiv (n-G)\,/G$ which is decreasing in G. In addition, ϕ is decreasing, thus ϕ^{-1} is decreasing as well.

Equation (19) gives a root of (20). Note that $G^* > 0$ which is obvious if $a \geq b$ and follows, if $a < b$, from

$$\begin{aligned}(a-b)\,n + \sqrt{n\left(4ab + n\,(b-a)^2\right)} &> (a-b)\,n + \sqrt{nn\,(b-a)^2} \\ &= n\,((a-b) + |b-a|) = 0\end{aligned}$$

A similar consideration shows that the second root of (20) is negative. ■

The optimal number of groups is not necessarily unique. The number given in the proposition is always welfare maximizing. However, if the technology is very productive, different organizational structures can lead to zero conflict in equilibrium and thus be optimal. This should be kept in mind in the discussion below.

The optimal number of groups has the following properties. First, typically the extreme cases $G = 1$ and $G = n$ will not be optimal - this follows directly from equation (20). Note that in the model the production technology gives no reason for dividing individuals in different groups. The reason for having several groups comes only from the rent-seeking activities. The findings are in sharp contrast to the case of a ratio type contest success function explored in Münster (2005) where the optimal number of groups is either 1 or n.

Second, if $a = b$, then $G^* = \sqrt{n}$. Since $n \equiv mG$, this means that there should be as many groups as there are individuals in any group.

Third, as one should expect, G^* is increasing in a and decreasing in b. (This is also true in a bang-bang sense in the case with a ratio-type contest success function.)

Fourth, G^* is increasing in n, when the total number of individuals is increasing the optimal number of groups also increases.

Fifth, the gains due to having the right structure can be quite substantial. In order to measure the gains, we compare the worst possible number of groups with the best possible number of groups, and calculate the percentage increase in productive effort. For simplicity, consider the case where $a = b$ and the production function has a constant elasticity form, $f(z) = z^h$ where $h > 0$ is a returns to scale parameter. Here, with an optimal number of groups, productive effort of an individual is $h/\left(b\left(n - \sqrt{n}\right)\right)$. On the other hand, the worst possible structure of the organization would be to have only one group, or equivalently n groups. In this case, productive effort is $h/\left(b\,(n-1)\right)$. Changing from the worst situation to the optimal number of groups, we find that the effort increases by the factor $1 + 1/\sqrt{n}$. The corresponding increase in output depends on returns to scale, output increases by the factor $\left(1 + \frac{1}{\sqrt{n}}\right)^h$. The gains can be quite substantial; e.g. if $n = 100$, productive effort increases by 10%. The gains are decreasing in n. They are maximal at $n = 4$ (for smaller n there is no way to make groups of equal size), where productive effort increases by 50%.

6 Conclusion

Many situations in economics and politics share the common structure that individual players belong to groups, and there is a potential for conflicts both within and between the groups. Examples include wars and civil wars as well as rent-seeking in multi-tiered organizations such as federal states, firms, or universities. This paper studies the interdependence of internal conflict, external conflict, and production. We set up an economic model of conflict and appropriation where players are partitioned in groups and can engage into appropriation both against the other groups and within their own group. We distinguish between a technology of external conflict, which describes how easy or difficult it is to take from the other groups, and a technology of internal conflict, which describes the possibilities for appropriation within groups. These technologies may differ because both the weapons used in the two types of conflicts as well as the institutions for conflict resolution differ.

In the basic model individual players decide simultaneously and independently about internal conflict, external conflict and production. We show that with a logistic technology of conflict, there is an extreme form of the 'group cohesion effect' that internal and external conflict are negatively correlated. Generically, there is only internal conflict, or only external conflict - but not both. Changes in the environment can switch the equilibrium from one type of conflict to the other. This may contribute to an explanation of the upsurge of internal wars in the former Eastern Block after the end of the cold war.

Decisions about external conflict are often taken in some political process rather than individually and independently. Thus we also study collective decisions, where groups decide about the amount of resources that each group member has to devote to the external conflict before individuals choose how to allocate their remaining resources to production and internal conflict.

Groups choose sufficiently high external conflict in order to avoid internal conflict. Thus, there is no internal fighting on the equilibrium path. If property gets more insecure *within* groups, and hence incentives to start fighting internally are high, this leads only to more *external* fighting. That remains true even if groups cannot gain much from the other groups by fighting against them. Thus, we propose a model of diversionary wars, which is, in contrast to the existing literature, not based on a principal agent problem.

The paper also has normative implications for the design of multi-tiered organizations like federal states or big firms. Even when the technology of production gives no reason for dividing individuals into groups, it is optimal to do so in order to minimize unproductive conflict. The optimal design of a multi-tiered organization is one where individuals are indifferent between putting resources in the internal or in the external conflict. Thus, having both types of conflict is optimal.

There are three directions for further research that we think would be particularly useful to explore. One is the targeting of external appropriation activities against specific groups, or of internal fighting against single individuals. A second topic is the endogenous formation and the stability of groups. These extensions seem particularly interesting if players are asymmetric ex ante. Finally, our model is static and thus one cannot study dynamics or repeated interaction, whereas existing models of dynamic conflict like Polborn (2005) and Bester and Konrad (2005) do not incorporate the distinction between internal and external conflict. Studying the interrelation between the two types of conflicts in a dynamic setting remains an important task.

A Appendix

A.1 Log-concavity of the objective functions

From (1), we have

$$\ln(u_{ig}) = \underbrace{b\sum_{j=1}^{m} y_{jg} - \ln\left(\sum_{k=1}^{G}\exp\left(b\sum_{j=1}^{m} y_{jg}\right)\right)}_{A(y_{ig})} +$$

$$\underbrace{ax_{ig} - \ln\left(\sum_{j=1}^{m}\exp\left(ax_{jg}\right)\right)}_{B(x_{ig})} +$$

$$\underbrace{\ln\left(f\left(\sum_{k=1}^{G}\sum_{j=1}^{m}(1 - x_{jk} - y_{jk})\right)\right)}_{C(x_{ig},y_{ig})}$$

To show that this is strictly concave in (x_{ik}, y_{ik}) look at the three terms in turn.

First look at $A(y_{ig})$. Let $V := \sum_{k\neq g}^{G}\exp\left(b\sum_{j=1}^{m} y_{jk}\right)$ for notational convenience. Then

$$A(y_{ig}) = by_{ig} + b\sum_{j\neq i} y_{jg} - \ln\left(V + \exp\left(by_{ig} + b\sum_{j\neq i} y_{jg}\right)\right).$$

Differentiating with respect to y_{ig}, we get

$$A'\left(y_{ig}\right) = b - b\frac{\exp\left(by_{ig} + b\sum_{j\neq i} y_{jg}\right)}{V + \exp\left(by_{ig} + b\sum_{j\neq i} y_{jg}\right)}.$$

This implies $A''\left(y_{ig}\right) < 0$, hence $A(y_{ig})$ is strictly concave in y_{ig}. In addition, note that $A(y_{ig})$ does not depend on x_{ig}.

Now look at $B(x_{ig})$. For notational convenience let $W := \sum_{j\neq i}^{m}\exp\left(ax_{jg}\right)$.

Then

$$B(x_{ig}) = ax_{ig} - \ln\left(\exp\left(ax_{ig}\right) + W\right),$$
$$B'(x_{ig}) = a - a\frac{\exp\left(ax_{ig}\right)}{\exp\left(ax_{ig}\right) + W}.$$

This implies $B''(x_{ig}) < 0$ and hence $B(x_{ig})$ is strictly concave in x_{ig}. In addition, note that $B(x_{ig})$ does not depend on y_{ig}.

Finally, look at $C(x_{ig}, y_{ig})$. By assumption, $\ln f(z)$ is strictly concave in z. This implies that

$$\frac{\partial^2}{\partial x_{ig}^2} C(x_{ig}, y_{ig}) = \frac{\partial^2}{\partial y_{ig}^2} C(x_{ig}, y_{ig}) = \frac{\partial}{\partial x_{ig}} \frac{\partial}{\partial y_{ig}} C(x_{ig}, y_{ig}) =$$
$$= \phi'\left(\sum_{k=1}^{G}\sum_{j=1}^{m}(1 - x_{jk} - y_{jk})\right) < 0.$$

The determinant of the Hessian Matrix is zero. Thus $C(x_{ig}, y_{ig})$ is weakly concave in (x_{ig}, y_{ig}).

Finally, we can put things together to show that $\ln(u_{ig})$ is strictly concave in (x_{ig}, y_{ig}). Write $\ln(u_{ig}(x, y)) = A(x) + B(x) + C(x, y)$. For any $(x, y), (x', y') \in \mathcal{R}_+^2$ and any $t \in (0, 1)$ we have

$$\begin{gathered}
\ln\left(u_{ig}(tx + (1-t)x', ty + (1-t)y')\right) = \\
= A\left(ty + (1-t)y'\right) + B\left(tx + (1-t)x'\right) + C\left(tx + (1-t)x', ty + (1-t)y'\right) < \\
< tA(y) + (1-t)A(y') + tB(x) + (1-t)B(x') + tC(x, y) + (1-t)C(x', y') = \\
= t\ln\left(u_{ig}(x, y)\right) + (1-t)\ln\left(u_{ig}(x', y')\right).
\end{gathered}$$

Hence $\ln\left(u_{ig}(x_{ig}, y_{ig})\right)$ is strictly concave in (x_{ig}, y_{ig}).

A.2 Proof of lemma 5

Suppose to the contrary that there are two individuals in the same group who choose different amounts of internal rent seeking. Without loss of generality, suppose

$$0 \leq x_{1g} < x_{2g} \leq 1 - y_g.$$

Since $x_{1g} < 1 - y_g$, from the first order condition for player 1 (see lines (9) and (10)), we have

$$\frac{a\sum_{j\neq 1}\exp(ax_{jg})}{\sum_{j=1}^{m}\exp(ax_{jg})} \leq \phi\left(n - m\sum_{k=1}^{G} y_k - \sum_{k=1}^{G}\sum_{j=1}^{m} x_{jk}\right).$$

On the other hand, since $0 < x_{2g}$, from the first order conditions for player 2 (lines (10) and (11)) we get

$$\phi\left(n - m\sum_{k=1}^{G} y_k - \sum_{k=1}^{G}\sum_{j=1}^{m} x_{jk}\right) \leq \frac{a\sum_{j\neq 2}\exp(ax_{jg})}{\sum_{j=1}^{m}\exp(ax_{jg})}.$$

Putting things together, we have

$$\sum_{j\neq 1}\exp(ax_{jg}) \leq \sum_{j\neq 2}\exp(ax_{jg})$$

or $x_{1g} \geq x_{2g}$, a contradiction.

A.3 Proof of lemma 6

Existence: The condition of the lemma (i.e. inequality (14)) implies that the first order condition (9) for the case with no internal fighting holds for all individuals. *Uniqueness*: Suppose there exists an equilibrium where $x_{ig} > 0$ for some individual i in a group g. Then, by condition (14) of the lemma, and (13), we have $\frac{\partial v_{ig}}{\partial x_{ig}} < 0$. This contradicts the first order conditions (9)-(11). *Payoffs* follow by inserting.

A.4 Proof of lemma 7

Existence: By (13), if equation (16) holds, we have

$$\frac{\partial v_{ig}}{\partial x_{ig}} = 0.$$

It remains to show that there exist $x_1, ..., x_G$ such that $0 \leq x_g \leq 1 - y_g$ for $g = 1, ..., G$ and such that equation (16) holds. Note that if $x_g = 1 - y_g$ for all $g = 1, .., G$ we have

$$\phi\left(n - m\sum_{k=1}^{G}(y_k + x_k)\right) = \phi(0) = \infty > \frac{a(m-1)}{m}. \qquad (21)$$

On the other hand, if $x_g = 0$ for all $g = 1, .., G$ we have

$$\phi\left(n - m\sum_{k=1}^{G}(y_k + x_k)\right) = \phi\left(n - m\sum_{k=1}^{G} y_k\right) < \frac{a(m-1)}{m}$$

since in this case the condition of the lemma, inequality (14), does not hold. By continuity of ϕ, there thus exist $x_1, ..., x_G$ such that $0 \leq x_g \leq 1 - y_g$ for $g = 1, ..., G$ and such that equation (16) holds. This completes the existence proof.

Uniqueness: Because of lemma 5 we only have to consider within-group symmetric equilibria. Equation (16) is equivalent to

$$m\sum_{k=1}^{G} x_k = n - m\sum_{k=1}^{G} y_k - \phi^{-1}\left(a\frac{m-1}{m}\right). \qquad (22)$$

To ease notation, let

$$X := n - m\sum_{k=1}^{G} y_k - \phi^{-1}\left(a\frac{m-1}{m}\right)$$

denote the right hand side of (22). Note that $X > 0$ since, by assumption, (14) does not hold. Using the notation just introduced, (16) is equivalent to $m\sum_{k=1}^{G} x_k = X$. Moreover, by (13), (16) is also equivalent to $\frac{\partial v_{ig}}{\partial x_{ig}} = 0$.

Suppose equation (16) does not hold. There are two possibilities. First, if $m\sum_{k=1}^{G} x_k < X$ we have $\frac{\partial v_{ig}}{\partial x_{ig}} > 0$ by (13). Then the first order condition implies $x_g = 1 - y_g$ for all groups, hence $m\sum_{k=1}^{G} x_k = n - m\sum_{k=1}^{G} y_k > X$, contradicting the assumption $m\sum_{k=1}^{G} x_k < X$. Second, if $m\sum_{k=1}^{G} x_k > X$ we have $\frac{\partial v_{ig}}{\partial x_{ig}} < 0$. Then the first order condition implies $x_g = 0$ for all groups, and hence $m\sum_{k=1}^{G} x_k = 0 < X$, contradiction. Hence, there are no equilibria where (16) does not hold.

Payoffs follow from inserting.

A.5 Proof of proposition 2

As argued in the main text, the fact that (17) is increasing in y_g implies that there is no internal fighting on the equilibrium path.

A.5.1 Case NO

Since $\phi(n) \geq a(m-1)/m$, inequality (14) holds for any possible $y_1, ..., y_G$. Hence in the second stage we have $x_{ig} = 0$ for all i and g, no matter what was chosen in stage 1. Therefore on stage 1 the groups solve

$$\frac{\exp(bmy_g)}{\sum_{k=1}^{G} \exp(bmy_k)} \frac{1}{m} f\left(n - m\sum_{k=1}^{G} y_k\right) \rightarrow \max_{0 \leq y_g \leq 1}.$$

Given $\phi(n) \geq b(G-1)/G$, the solution of this game is $y_1 = ... = y_G = 0$.

A.5.2 Case INTRA

To begin with, note that equation (18) defines a unique y, namely

$$y = 1 - \frac{1}{n}\phi^{-1}\left(\frac{a(m-1)}{m}\right) \in (0,1) \tag{23}$$

by the same argument as in the proof of lemma 2.

Existence. If all groups choose $y_g = y$ given in (23), then in stage 2 $x_{ig} = 0$ for all i and g. Fix all $y_2 = ... = y_G = y$ and think of u_{i1} as a function of y_1. Choosing a $y_1 < y$ leads to internal conflict in stage 2, and is thus never optimal - in this range u_{i1} is strictly increasing in y_1 (see line (17) in lemma 7). On the other hand, if $y_1 \in [y, 1]$ there is no internal fighting. In this range, the objective function u_{i1} is

$$u_{i1} = \frac{\exp(bmy_1)}{\exp(bmy_1) + (G-1)\exp(bmy)} \frac{1}{m} f(n - (G-1)my - my_1) \tag{24}$$

(see line (15)). We will show that u_{i1} is strictly decreasing in y_1 for all $y_1 > y$. From (24),

$$\frac{\partial u_{i1}}{\partial y_1} < 0 \text{ iff } b\frac{(G-1)\exp(bmy)}{\exp(bmy_1) + (G-1)\exp(bmy)} < \phi(n - (G-1)my - my_1).$$

For all $y_1 > y$,

$$\begin{aligned}
b\frac{(G-1)\exp(bmy)}{\exp(bmy_1)+(G-1)\exp(bmy)} &< b\frac{G-1}{G} \\
&< a\frac{m-1}{m} \\
&< \phi\left(n-(G-1)my-my_1\right)
\end{aligned}$$

where the second inequality is from (INTRA), and the third inequality is from equation (18), $y_1 > y$ and $\phi' < 0$. Hence for all $y_1 > y$, u_{i1} is strictly decreasing in y_1, and group 1 has no incentive to increase y_1 over y.

Uniqueness. Here we argue that there is no symmetric subgame perfect equilibrium where y is not as given by (23).

Suppose

$$y < 1 - \frac{1}{n}\phi^{-1}\left(\frac{a(m-1)}{m}\right).$$

Then we have internal fighting, contradiction. On the other hand, suppose

$$y > 1 - \frac{1}{n}\phi^{-1}\left(\frac{a(m-1)}{m}\right).$$

Then we have $\phi(n(1-y)) > a\frac{m-1}{m} > b\frac{G-1}{G}$. Hence each group would gain from choosing a slightly lower y_g - this still leads to zero internal conflict in the second stage, and the lower share of group g is outweighed by the corresponding increase in output.

A.5.3 Case INTER

Existence. As in lemma 3, equation (7) defines a unique $y \in (0,1)$. If $y_1 = ... = y_G = y$ then we have $x_{ig} = 0$ for all i and g. Fix all $y_g = y$, $g = 2, ..., G$ and think of u_{i1} as a function of y_1. Choosing a y_1 such that there is internal fighting can never be optimal. On the other hand, among the range where there is no internal fighting, u_{ig} is a concavifiable objective function to be maximized over a convex set, and at the proposed value the derivative of the objective function is zero.

Uniqueness. Here we argue that there is no symmetric subgame perfect equilibrium where y is not given by equation (7). Suppose that

$$y < 1 - \frac{1}{n}\phi^{-1}\left(b\frac{G-1}{G}\right).$$

Then each group would profit from increasing y_g.

On the other hand, if

$$y > 1 - \frac{1}{n}\phi^{-1}\left(b\frac{G-1}{G}\right)$$

each group would profit from decreasing its y_g.

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