Vote Buying and the Education of a Society

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Abstract
Various studies provide evidence that buying of votes is a widespread instrument of parties in developing countries to influence the outcome of elections. In this paper we examine whether democratic societies can escape poverty traps via redistribution if vote buying is possible. We show that if agents can buy or sell votes, then the education of a society is impossible because human capital enhancing redistribution will not occur. Hence, society will remain in the poverty trap. We then show that the negative effects of vote buying can be eliminated by means of a repeated voting constitution in combination with a threshold flexible majority voting rule or a constitution with a threshold flexible minority voting rule and repeated subsidization.

Keywords: vote buying, political economy, poverty traps, economic development, flexible majority rules, repeated voting

JEL: C72, D72, O10, P16.

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1 Introduction

There is a controversial theoretical and empirical discussion as to whether or not democracies may lead to economic growth.\(^1\) In this context, Gersbach and Siemers (2005) have shown that a democratic society with appropriate democratic rules may overcome poverty via redistribution. However, even when a democracy is well established redistribution might be problematic because parts of a society may have incentives to undermine existing rules. One possible instrument to prevent or to foster redistribution is vote buying.\(^2\) Various studies provide evidence that buying of votes is a widespread instrument of parties in developing countries to influence the outcome of elections. For example, in countries like Mexico, Philippines, Senegal, Taiwan or Thailand buying of votes has a long tradition. In the Philippines 2002 (community-level) elections, an estimated 3 million people were offered some form of payments which corresponds to about 7 percent of all voting-aged adults. In Thailand, 30 per cent of households heads surveyed in a national sample said that they were offered money during the 1996 general election. In Taiwan’s 1999 election, 27 per cent of a random sample of eligible voters reported that they had accepted cash offers during previous electoral campaigns.\(^3\)

However, if vote buying is present, then the success of redistribution policies used to overcome poverty may possibly be endangered. There exists theoretical and empirical evidence which supports this view. Buchanan and Tullock (1962) argue that minority groups, for example the poor, are likely to face higher transaction costs in vote markets than others and may therefore become the victims of income redistribution. Aghion and Bolton (2003) assert the fear of Schelling (1960) that vote trading tends to increase the scope for expropriation of voters. Particularly with regard to economic growth, Docquier and Tarbalouti (2001) analyze the potential effects of vote buying on redistribution in developing countries. They argue that some (rich) groups may have incentives to buy votes to prevent excessive redistribution. Their main result is that vote buying is likely to reduce redistribution and growth. Anderson and Tollison (1990) argue that prior to the introduction of secret ballots towards the end of the nineteenth and beginning of the twentieth century, (illegal) buying of votes was widespread in

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\(^2\)In the literature different notions of “vote buying” are discussed. For an overview see, for example, Schaffer (2002b). In this paper, vote buying is seen as a purely economic exchange, where votes are traded for example for cash.

\(^3\)See, for example, Hicken (2002), Rigger (2002), Schaffer (2002a), Schaffer (2004) and Diaz-Cayeros et al. (2003).
Britain and the United States of America. They assert that the elimination of vote buying had a non negligible effect on the historical rise in government expenditures on redistributive policies.

The objective of this paper is to analyze the effects of vote buying on the economic development of a democratic society which is caught in a poverty trap. Therefore, we use the political economy model developed by Gersbach and Siemers (2005). We show that if vote buying is possible in this setting, then the education of a society is impossible and hence society will be caught in a poverty trap further on unless additional constitutional rules are established. We then show that repeated voting constitutions or constitutions with a threshold flexible minority voting rule may be appropriate to eliminate the potential negative effects of vote buying.

The paper is organized as follows. In the next section we give a survey of the related literature. In section 3 we present the basic model. In section 4 we discuss the necessary policy to educate a society. In section 5 we outline the political framework. In section 6 we present the vote buying model. In section 7 we first show that if vote buying is possible then the education of a society may fail, then we present appropriate constitutional rules to eliminate the potential negative effects of vote buying. Section 8 concludes.

2 Related Literature

Our paper is related to different strands of the literature. The first strand refers to the literature about child labor. There is a large and rapidly growing range of theoretical and empirical literature on this subject. For an overview, see, for example, Basu and Tzannatos (2003) and Jafarey and Lahiri (2001).4

The second strand refers to the political economy literature focusing upon redistribution policies. For an overview see, for example, Persson and Tabellini (1997), Persson and Tabellini (2000) or Drazen (2000). However, most of these analyses deal with transfers from young to old in the social-security context. In contrast to our work, Acemoglu and Robinson (2001), amongst others, have shown that a democracy may not consolidate because it is more redistributive than a non democratic regime, and this gives the elite an incentive to mount a coup. However, the main concern of this

4A more detailed theoretical discussion of this issue is given, for example, in Basu (1999), Basu (2000), Basu and Van (1998), Bell and Gersbach (2001), Dessy (2000), Swinnerton and Rogers (1999).
paper is about the effects of vote buying on redistribution within a dynamic political process and the constitutional design to eliminate potential negative effects of vote buying.

There is a growing range of literature on vote buying.\(^5\) For example, Buchanan and Tullock (1962) provide a theoretical rationale for the prohibition of vote buying (see Buchanan and Tullock (1962), pages 270-274). Their focus is on the efficiency of vote trading. They argue that under the unanimity rule, free trade in votes would lead to an efficient outcome. Under the simple majority rule, they assert that an efficient outcome may only be possible if there exists a perfect market for votes.\(^6\) However, they claim that market imperfections are very likely, i.e., vote buying would lead to an inefficient result. Kochin and Kochin (1998) emphasize the costs of buying votes and forming blocking coalitions in political elections. These costs may lead to inefficient decisions which depend on the source of the cost and their distribution. In a model with two competing parties, Groseclose and Snyder (1996) analyze a vote buying game where parties move sequentially and only once. One of their main results is that the second mover has a substantial advantage. They show that if the voters are indifferent between the proposals of both parties, the first mover needs a sufficiently high budget to bid in such a way that the second mover cannot afford to buy any majority.

Dekel et al. (2004) examine in a sequential and alternating bidding process the consequences of vote buying. The main focus of their analysis is on the role of parties' budget constraints and voter preferences in determining the winner and the payment to voters. They distinguish between up-front payments (payments that are contingent on the vote, but not on the outcome) and campaign promises (payments that are contingent on the outcome of the election, but not on the actual vote). One of their results is that the payments received by the voters under campaign vote buying tend to be substantially higher than under up-front vote buying. The intuition is that, with up-front vote buying the party that knows it will lose in equilibrium will not wish to buy any votes since they must be paid up-front, but it will be happy to bid for votes with campaign promises that are only paid upon winning. In contrast to Groseclose and Snyder (1996), the symmetric bidding process of Dekel et al. (2004) neutralizes the effect of the order of moves and therefore the party with the highest budget will win the vote buying game.

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\(^5\)For an overview see Grossman and Helpman (2001).

\(^6\)In this context, Philipson and Snyder (1996) examine an organized market for votes, in which trade is directed by an intermediary. They show that the outcome with vote trading is Pareto superior to the outcome without vote trading.
Dal Bó (2000) analyzes the effects of vote buying of an outside party on the collective decisions made through voting in a committee. He shows that vote buying induces inefficient outcomes and that this result is very robust with regard to, for example, changes in payoff and information structures or different incentive schemes. For this reason, he develops protective measures for committees. He, amongst others, argues that granting the committee the authority to change its own decision rules can reduce the susceptibility of committees to vote buying. Moreover he discusses the implications of vote bribing for votes in legislatures and for the efficiency of democracy.

There are also closely related models which concentrate on lobbying. Dharmapala (1999) combined the legislative bargaining model of Baron and Ferejohn (1989) with models of interest lobbying (such as the common agency models developed by Grossman and Helpman (1996) and Dixit et al. (1997)). One of their main results is that, when lobbying occurs, a supermajority coalition may, under quite general circumstances, be less expensive than a minimal coalition.

Another strand of the literature refers to the constitutional economics. Our constructive constitutional economics approach goes back to Buchanan and Tullock (1962). Recent papers on constitutional design focus on optimal majority rules in the context of reforms and public goods provision. For example, Erlenmaier and Gersbach (1999) propose a so-called flexible majority rule for public goods provision where the required majority depends upon the proposal itself. In this paper, we apply flexible majority rules to redistribution proposals.

## 3 The Basic Model

As mentioned in the introduction, we extend the model of Gersbach and Siemers (2005) who embed the model of Bell and Gersbach (2001) in a political-economic setting. Following these authors, we assume that human capital accumulation is the main source of economic growth. A description of the underlying basic model is given in the following subsections.

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7For an overview, see Grossman and Helpman (2001).
3.1 The Structure of Society

In the following, we consider an OLG model in which individuals live for two periods. These periods are labeled childhood and adulthood, respectively. Each household comprises one adult and one child.

We now describe the structure of the society. We consider a society $\Omega = \{1, \ldots, n\}$ consisting of $n > 3$ groups of individuals, representing, for example, interest groups, ethnic groups, parties, and so on. This assumption seems plausible for developing countries. The members of a group are assumed to be homogeneous and they are treated identically. That is, all members of each group will have the same amount of human capital. Each group is indexed by $i$ and consists of a measure of $\Delta_i$ individuals, where

$$\Delta_i = \Delta = \frac{1}{n} \forall i \in \Omega$$

(1)

That is, we assume that the groups are of equal size and each generation consists of a continuum of households represented by the interval $[0, 1]$.\(^{10}\)

3.2 Human Capital Formation and Output Production

We now turn to the formation of human capital. Consider a member of group $i$ in period $t$. Human capital is assumed to be formed in a process that combines child-rearing with formal education in the following way: Let $\lambda_i^t$ denote the adult’s endowment of human capital. In the course of rearing a child, an activity that is assumed to take up a fixed amount of time, the adult gives the child a certain capacity to build human capital for adulthood. The amount of this contributing factor is assumed to be a fixed fraction, $z \in (0, 1]$, of the adult’s own endowment of efficiency units of labor. Let the portion of childhood devoted to education in period $t$ be denoted by $e^t_i \in [0, 1]$, the residual portion being allocated to work. Adults spend all their time working. Based on these assumptions, the child’s endowment of efficiency units of labor on reaching adulthood at time $t + 1$ is given by

$$\lambda_{i+1}^t = h(e^t_i) z \lambda_i^t + 1.$$  

(2)

\(^{9}\text{For example, Moser (2004) provides some evidence for Madagascar that ethnic group affiliation might play a role in voting behavior.}\)

\(^{10}\text{The assumptions are made for the sake of simplicity. Note that for } n \to \infty, \text{ the number of groups will coincide with the number of individuals in the society. This case would correspond to an atomic society.}\)
The function \( h(\cdot) \) represents the educational technology. \( h(\cdot) \) is assumed to be a continuous, increasing and differentiable function on \([0, 1]\), with \( h(0) = 0 \). Equation (2) implies that rearing and formal education are both necessary if human capital is to be formed at all in the next generation, i.e., if \( \lambda^i_{t+1} > 1 \). The condition \( \lambda^0_i = 1 \ \forall i \in \Omega \) can be thought of as state of backwardness.

We turn now to the technology for producing output. We consider an aggregate consumption good. Let us assume that output is proportional to inputs of labor, measured in efficiency units. Wages are paid according to marginal productivity, whereby all output will accrue to the household as income. Following Bell and Gersbach (2001), without any education, the child will supply at best \( \gamma \) efficiency units of labor, where \( \gamma \in (0, 1) \). The household therefore supplies a total of \([\lambda^i_t + (1 - e^i_t)\gamma]\) efficiency units of labor to the production of the aggregate good. Hence, the level of output produced by a household that has an endowment of \( \lambda^i_t + \gamma \) and chooses \( e^i_t \) is

\[
y_t = \alpha [\lambda^i_t + (1 - e^i_t)\gamma], \tag{3}
\]

where \( \alpha \in (0, \infty) \) denotes the productivity of human capital measured in efficiency units of labor.

### 3.3 The Household’s Behavior

We assume that all allocative decisions lie in the adults’ hands.\(^{11}\) We rule out any bequests at death and the possibility of debts, so that (3) is the current real income and is consumed completely. Since current consumption is maximized by choosing \( e^i_t = 0 \), it follows that the adult’s sense of altruism towards her child must be sufficiently strong for her to choose \( e^i_t > 0 \).

Let \( c^i_t \) denote the consumption of a household of group \( i \). From (3) we then obtain the family’s budget constraint

\[
c^i_t + \alpha \gamma e^i_t = \alpha (\lambda^i_t + \gamma). \tag{4}
\]

With this preliminary settled, the adult’s preferences for consumption and education can be summarized by the optimal choices of \( e \) and \( c \), denoted by \( e^0(\lambda_t) \) and \( c^0(\lambda_t) \), which are continuous in \( \lambda_t \).\(^{12}\) There are two threshold values, \( \lambda^S \) and \( \lambda^a \), which depend on the preferences of the households. As long as \( \lambda_t \leq \lambda^S \), \( e^0(\lambda_t) = 0 \), and as long as


\(^{12}\)A deeper foundation of preferences of adults is given in Bell and Gersbach (2001).
\[ \lambda_t \geq \lambda^a, \ e_t^o(\lambda_t) = 1. \] We assume that both goods are non-inferior and therefore that

\[ \frac{\partial e^o}{\partial \lambda_t} > 0 \quad \text{and} \quad \frac{\partial e^o}{\partial \lambda_t} > 0 \]

for \( \lambda_t \in [\lambda^S, \lambda^o] \).

### 3.4 The Dynamics

Given the assumptions made in subsection 3.3, we now return to equation (2). We obtain:

\[
\lambda_{t+1} = \begin{cases} 
1 & \forall \lambda_t \leq \lambda^S; \\
zh(e_t^o(\lambda_t))\lambda_t + 1 & \forall \lambda_t \in (\lambda^S, \lambda^o); \\
zh(1)\lambda_t + 1 & \forall \lambda_t \geq \lambda^a.
\end{cases}
\] (5)

The following qualitative results are immediately obtained: In view of the assumption that \( \lambda^S > 1 \), it follows from the first part of (5) that the state of backwardness, \( \lambda = 1 \), is a locally stable equilibrium. Henceforth, we will refer to this equilibrium as the poverty trap. In the following, we concentrate on the growth case, where \( zh(e_t^o) > 1 \) for \( e_t^o \in (0,1] \) and \( \lambda^* < \lambda^o \). Thus, in this case, the dynamic system has exactly two steady states, namely \((\lambda^*, e^o(\lambda^*))\) and \((1,0)\), where the former is unstable. The dynamic of our model is simplified in figure 1.

Figure 1 shows that sustainable education and hence long-term growth crucially depend on the educational choice of the adults of a group which, in turn, depends on their human capital level. For example, if the education of the children was not sufficient, i.e. \( e_t^o < e^o(\lambda^*) \), then the human capital of these children and the human capital of their lineages in the following periods will be smaller than \( \lambda^* \). That is, the education will not suffice for a permanent increase in human capital and, accordingly, the offspring of this group will fall back into the poverty trap. However, if the adults of a group choose \( e_t^o > e^o(\lambda^*) \), then the human capital of the descendants of this group in the next and in the following periods will be greater than \( \lambda^* \). Correspondingly, in this case education will suffice for a sustainable increase in human capital and hence the offspring of this group will leave poverty behind.

To sum up: In order to permanently overcome the poverty trap, uneducated groups have to be supported sufficiently so that the adults of this group can afford sustainable schooling, \( e_t^o > e^o(\lambda^*) \), for their offspring.
4 Policies to Overcome Poverty

In this and the following sections we assume that the whole society is initially \((t = 0)\) in the state of backwardness, i.e. \(\lambda^*_0 = 1\ \forall i \in \Omega\). The broad policy objective is to educate the whole society to enable all its members to escape from this backwardness. The instruments for this purpose are taxation and subsidization.

We assume that only the income of adults is subject to taxation. Let \(\tau^i_t\) denote the tax levied in period \(t\) on the income of an adult in group \(i\). Recall that all adults in group \(i\) are assumed to be identical. That is, each member of group \(i\) has the same income and therefore, each adult has to pay the same amount of taxes. Hence, the total amount of taxes of group \(i\) amounts to \(\Delta \tau^i_t\). At the beginning of each period \(t\), some groups will be subsidized from the ensuing tax revenue. We use \(s^i_t\) to denote the subsidy each household in group \(i\) will receive in period \(t\). Accordingly, group \(i\) will receive subsidies amounting to \(\Delta s^i_t\).

We define \(\overline{\sigma}\) as the subsidy a household in a state of backwardness needs in order to achieve a human capital level of \(\lambda^* + \epsilon\ (\epsilon > 0)\) in the following period. Hence, for a household that is not taxed, \(\overline{\sigma}\) is given by the implicit equation:

\[
z h(e^\alpha (\alpha + \overline{\sigma})) + 1 = \lambda^* + \epsilon
\]  

(6)
For reasons of efficiency, we assume that a household is either taxed or subsidized. Since households in a state of backwardness have few resources, we assume that there is a subsistence level \( c_{sub} \) for an adult-child household which must be ensured under all circumstances. The taxation of a household of group \( i \) caught in the poverty trap is therefore assumed to be constrained by:

\[
\alpha \lambda_t^i - \tau_t^i + \alpha \gamma \geq c_{sub}
\]

In particular, the tax must fulfill the following condition:

\[
\tau_t^i \leq \alpha (1 + \gamma) - c_{sub} : = \tau^{sub}
\]  

where it is plausible that \( \tau^{sub} \) is small, since households with \( \lambda_t = 1 \) may already be close to the subsistence level \( c_{sub} \).

We now turn to the households of a group \( i \) which have received subsidies \( s_t^i \geq \bar{s} \). In order to limit the taxation of former subsidized and now educated households so that the education achieved in the period of transfer is sustainable, the tax must fulfill the following condition:

\[
\tau_t^i \leq \alpha \left( z h \left( c_t^i (\alpha + \bar{s}) \right) + 1 - (\lambda^* + \epsilon) \right) : = \tau^*
\]

The total government revenues in period \( t \) are denoted by \( B_t \). The budget constraint in a period \( t \) is given by:

\[
B_t = \sum_{i=1}^{n} \Delta \tau_t^i \geq \sum_{i=1}^{n} \Delta s_t^i
\]

Throughout the paper, we assume that the maximum resources that can be levied through taxation are sufficient to finance schooling of the children of at least one group, i.e., \( B_t \geq \Delta \bar{s} \).\(^\text{13}\)

5 The Constitution

We consider the case of secret ballots in a direct democracy with a voting population consisting of the parent-generation.\(^\text{14}\) At the constitutional stage, we assume that

\(^\text{13}\)This assumption is made solely to simplify the analysis. Note that Gersbach and Siemers (2005) have shown that there are democratic rules, e.g., repeated agenda setting, which guarantee that a society will be educated if this condition does not hold.

\(^\text{14}\)Helpman (1995) stresses that although direct democracy is rarely applied, majority voting via direct democracy is a good approximation for outcomes in representative democracy as the results are reasonably close.
individuals decide behind a veil of ignorance how to educate the society. Since educating
the society leads to long-term growth and investing in basic education is obviously
socially profitable, every lineage can be generally better off over time. Hence, behind
a veil of ignorance, the individuals unanimously agree on constitutional principles that
ensure education for the society as a whole. We now consider a constitution and the
corresponding constitutional rules developed by Gersbach and Siemens (2005), which
enable a society to escape from a poverty trap if vote buying is not possible:

**Agenda Setting Rule:**

The first crucial step is the rule by which the agenda setter is chosen. We specify the
democratic agenda setting process as follows:

- **Rotating agenda setting (RoA):** In the first period each group \( i \) has the same
  chance to make a proposal. In the subsequent periods, only groups which have
  not set the agenda in the previous periods can apply for agenda setting. Each
  group \( i \) which is allowed to make a proposal has the same chance to set the
  agenda.

This rule implies that the number of allowed reelections is restricted, possibly to zero.
That is, each group \( i \) is only allowed to set the agenda once within \( n = \frac{1}{N} \) periods. It
follows that the only period in which a household can expect to enjoy a subsidy is the
period in which its group has been selected to determine the agenda. It is plausible for
the agenda setting group to distribute tax revenues equally among its members, and
we will assume this in the following.\(^{15}\)

**Agenda Rule:**

We restrict the set of allowed proposals to one that satisfies the governmental budget
constraint with the agenda rule BB:

- **Balanced budget (BB):** A constitutional proposal has to satisfy a balanced bud-
  get, i.e.,

\[
\sum_{i=1}^{n} \Delta x^i_t - \sum_{i=1}^{n} \Delta y^i_t = 0, \quad \forall t
\]

**Decision Rule:**

We now turn to the decision rule. We apply the *threshold flexible majority rule* introduced by Erlenmaier and Gersbach (1999) and Gersbach (2004) for the provision of

\(^{15}\) We do not explicitly analyze how the group decides upon an agenda; it suffices to know that they
will maximize tax revenues for the group and divide them equally.
public goods. In our context, we will use flexible majority rules to limit the taxation of educated households so that they do not fall back into poverty. We define

$$\tau_t^{max} = \max_{i \in \{1, \ldots, n\}} \tau_t^i.$$  

- **Threshold flexible majority rule (TFM[\tau_t^{max}, \overline{\tau}]):** Under this rule, the share of votes needed to implement a proposal, \( m(\tau_t^{max}, \overline{\tau}) \), jumps from \( \frac{1}{2} \) to 1 if any group \( i \) is taxed higher than the threshold tax \( \overline{\tau} \) stated in the constitution:

$$m_i(\tau_t^{max}, \overline{\tau}) = \begin{cases} \frac{1}{2} & \text{if } \tau_t^{max} \leq \overline{\tau}; \\ 1 & \text{if } \tau_t^{max} > \overline{\tau}. \end{cases}$$

The threshold flexible majority rule ensures that a winning majority for the proposal denoted by \( P_t = \{ \Delta \tau_t^i, \Delta s_t^i \}_{i=1}^n \) can be obtained if, and only if, educated adults are not taxed adversely, i.e. if \( \tau_t^{max} \leq \overline{\tau} \). Adverse taxation means that former subsidized and now educated groups are taxed most heavily because they can pay the highest taxes. This would imply that children of educated taxed households will be not educated at all. As soon as an agenda setter suggests an adverse tax scheme, the constitution requires unanimous agreement, which, de facto, makes such taxation impossible to implement. Note that the prevention of unconstitutional taxation must be enforced by a constitutional court.

To sum up, Gersbach and Siemers (2005) have shown that under the constitution \( C\{BB, RoA, TFM(\tau_t^{max}, \overline{\tau})\} \) democracies may be able to educate a society. However, there might be other political failures which could undermine the success of such a constitution. In the next section we consider the fact that each group may have incentives to buy or sell votes in order to influence the outcome of the voting. For example, it might be profitable for designated taxed groups to make direct payments to untaxed voters in order to influence them to vote against the implementation of the proposal.

6 The Model with Vote Buying

In the following, we assume that designated taxed groups may engage in up-front vote buying. Up-front vote buying is a binding agreement that gives a group full control of the vote of another group in exchange for an up-front payment. Hence, the agenda setter may have an incentive to pay subsidies to untaxed groups in order to defend its proposal against possible attacks of designated taxed groups.
So far, we have assumed that individuals within groups can coordinate themselves. Now suppose that coordination among different groups is also possible. That is, the designated taxed groups may have an incentive to form a coalition in order to prevent the implementation of the proposal by means of vote buying. For simplicity, we assume that each group in a coalition will have the same bargaining power, i.e. each group has to make the same amount of payments.

The basic structure of the vote buying game follows Groseclose and Snyder (1996). We assume a sequential game, with the agenda setting group, henceforth denoted by $as$, moving first and the coalition of designated taxed groups moving last. This assumption could be justified with the observation that, in reality, payments for votes on a bill are typically written into the bill itself, as for example special conditions, subsidies, exemptions, and so on. In our case, the payments of the agenda setting group for votes are part of the proposal, i.e., they are part of the bill, and if it is costly to change bills, then the coalition of the designated taxed groups is effectively able to move last. Note that, if the randomly chosen agenda setter will not apply for agenda setting, the status quo will prevail. Also note that individuals know at the vote buying stage who will be taxed and who will receive subsidies if a proposal is accepted. In particular, at this point in time the coalition of designated taxed groups is perfectly informed about the offers made by the agenda setting group.

We now turn to the sequence of the vote buying game. The timing of events in period $t$ can be summarized as follows:

1. A group is randomly chosen to set the agenda. The agenda setter either announces a project proposal and its financing scheme, $P_t = \{\Delta r_t, \Delta s_t\}_{i=1}^n$, or makes no proposal.
2. (a) If the agenda setter made no proposal, the status quo will prevail.
   (b) If the agenda setter announced a proposal, then
      (i) The designated taxed groups decide on the basis of this proposal whether or not to buy votes.
      (ii) Vote buying does or does not take place.
      (iii) The society holds a vote on the implementation of the proposal.
      (iv) The project is or is not adopted. If the project is not adopted, the status quo will prevail.
Given the constitution described in section 5, we will look at subgame perfect implementations in stages one and two. In order to find unique subgame perfect equilibria, we introduce two tie-breaking rules. As a tie-breaking rule, TR, for agenda setting, we assume:

- **TR 1**: Group $i$ will not apply for agenda setting if:
  $$s_i^i = 0$$

That is, the agenda setting group will announce a proposal, if and only if, it can strictly improve the utility of its members by agenda setting, i.e., if the subsidies that they will receive are positive. Alternatively, we can assume that there exist small but positive fixed costs of agenda setting.

Referring to the voting behavior of un bribed groups, we assume that voters of group $i$ obviously support proposal $P_i$ if $s_i^i > 0$ and reject it if $\tau_i^i > 0$. However, if $s_i^i = \tau_i^i$, then the voters of the un bribed group $i$ will be indifferent between supporting and rejecting the proposal. As a tie-breaking rule, we assume

- **TR 2**: A voter of un bribed group $i$ supports the proposal $P_i$ if:
  $$s_i^i = \tau_i^i$$

Recall that we have assumed that a proposal either levies taxes on individuals of a group (including a zero tax rate) or provides subsidies. That is, designated taxed groups will vote against the proposal, whereas designated untaxed groups which are unin bribed will vote in favor of the proposal. Note that we implicitly assume that the voting behavior of the voters is myopic, i.e., households will not take into account that their voting behavior may influence the outcome of votings of subsequent generations.

We now observe the vote buying stage. If vote buying occurs, both the agenda setting group and the coalition of designated taxed groups are interested in obtaining a majority of votes for and against the proposal, respectively, while spending as little as possible. That is, they compete for the votes of the groups determined not to be taxed. We now turn to the payment promises of the agenda setting group and of the coalition of taxed groups to the untaxed groups. We define $NT = \{ i \in \Omega \mid \tau_i^i = 0 \land i \neq as \}$ as the set of the designated untaxed groups in which the agenda setting group is not included. Formally, let $s_i^i$ denote the offer of the agenda setting group to each member of the untaxed group $i \in NT$, and let $p_i^i$ denote the payment offer of the coalition of designated taxed groups to each member of the untaxed group $i \in NT$.  

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In the following, we assume that both the agenda setting group and the coalition of designated taxed groups have an incentive to purchase untaxed groups if, and only if, the expected tax revenues, respectively the tax burden, denoted by $B_t$, are strictly higher than their total payment promises to the designated untaxed groups. However, if this is not the case, buying of votes will not occur. That is, we assume that agents will only buy votes if the gain from vote buying is positive. Alternatively, $B_t$ can be interpreted as the budget or willingness to pay for implementing and preventing the proposal, respectively, of the agenda setting group and the coalition of designated taxed groups.

The incentives of these groups can be formalized basically as follows:  

- **VB(1):** The agenda setting group will buy votes if, and only if:
  
  $$ B_t > \sum_{i \in NT} \Delta s^i_t, \quad \text{with} \quad s^i_t \geq 0 $$

- **VB(2):** A coalition of designated taxed groups will buy votes if, and only if:
  
  $$ B_t > \sum_{i \in NT} \Delta p^i_t, \quad \text{with} \quad p^i_t \geq 0 $$

The preference of the agenda setting group and the coalition of designated taxed groups is to win at minimal cost. In equilibrium, the utility of winning of the agenda setting group is $\Delta s^{as}_t = B_t - \sum_{i \in NT} \Delta s^i_t > 0$ and its utility of losing is zero, where $\Delta s^{as}_t$ is the subsidy for the agenda setting group and $\sum_{i \in NT} \Delta s^i_t$ the total of all payments incurred by the agenda setting group (including zero subsidies to some of the untaxed groups). In contrast, in equilibrium, the (dis)utility of winning for the coalition of designated taxed groups amounts to $-\sum_{i \in NT} \Delta p^i_t$ and its utility of losing is $-B_t$, where $\sum_{i \in NT} \Delta p^i_t$ is the total of all payments incurred by the coalition of designated taxed groups (including zero payment offers to some of the untaxed groups) and $B_t - \sum_{i \in NT} \Delta p^i_t > 0$ is the value of winning if the majority votes against the proposal in equilibrium. Note that the coalition of designated taxed groups will only engage in vote buying if it knows that it will win in equilibrium. Otherwise, the coalition of designated taxed groups will not buy any votes since those have to be paid for up-front. Also note that, if the proposal of the agenda setting group is rejected, then the agenda setting group does not have to fulfill its payment promises.

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16 We assume, similar to Dekel et al. (2004), that the behavior of the groups and their members respectively is motivated by considerations of utility maximization.
We now turn to the designated untaxed groups which may have incentives to sell their votes either to the agenda setting group or to the coalition of the designated taxed groups. We assume that voters of untaxed groups sell their votes according to the following rule:

- **VS**: A designated untaxed group \( i \in NT \) will sell the votes to the agenda setting group if, and only if:

\[
s_i^t \geq p_i^t
\]

VS states that the coalition of designated taxed groups has to promise a designated untaxed group more than the agenda setting group in order to win its vote.\(^{17}\) Note that we implicitly assume that a vote buyer can monitor the casting of votes of the purchased groups. That is, we assume that the purchased groups will not deviate and vote in favor of the opponent group. In reality, vote buyers can, for example, punish recipients if they do not vote as instructed or they can pay voters to abstain from voting.\(^{18}\) Additionally, we assume that the agenda setting group cannot observe which groups are bought by the coalition of the designated taxed groups. That is, we implicitly assume that interest groups can buy votes at negligible costs, the risk of vote buyers and sellers of being arrested and punished is zero and that there are no moral costs of vote buying. Many studies provide evidence that these assumptions seem plausible for developing countries.\(^{19}\) In the conclusions we discuss how these assumptions impact on our results.

7 Results with Vote Buying

7.1 The Impossibility Result

In the following we use \( T \) to denote the number of periods a democratic society needs to educate itself. Recall our assumption that the whole society is initially \( t = 0 \) in the state of backwardness, i.e. \( \lambda^t_0 = 1 \ \forall i \in \Omega \). If there are vested interests and buying of votes is possible, we obtain:

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\(^{17}\)Note that this assumption is merely made in order to simplify the analysis and the exposition.

\(^{18}\)Schaffer (2002b) gives a description of a number of strategies available to vote buyers to generate or reinforce compliance.

\(^{19}\)See, for example, Schaffer (2002b), Diaz-Cayeros and Magaloni (2003) and Pfeiffer (2004).
Proposition 1
Consider the case in which there is a democracy with $C\{BB, RoA, TFM(\tau_{i}^{\text{max}}, \overline{\tau})\}$, with $\overline{\tau} = \min\{\tau_{\text{sub}}, \tau^*\}$, where $\tau_{\text{sub}}$ is the highest taxation allowed for members of groups in a state of backwardness and $\tau^*$ is the highest tax burden for members of an already subsidized group that does not endanger schooling in the future. Such a democracy cannot educate a society in finite time, i.e. $T = \infty$, when vote buying is possible.

The formal proof is given in the appendix.

Proposition 1 states that, under the constitution with the threshold flexible majority rule (TFM), the balanced budget rule (BB) and the rotating agenda setting rule (RoA), the possibility of vote buying will not enable a society to escape from the poverty trap. Intuitively, the reason for this result is that the coalition of designated taxed groups will only make payment offers to untaxed groups when the agenda setter makes a proposal on which the society holds a vote. In order to win the vote, the agenda setting group has to make equal payment offers to all designated untaxed groups, since it knows that the coalition of designated taxed groups will engage in vote buying and the coalition of designated taxed will buy the “cheapest” untaxed groups if it has made different payment offers across groups. In contrast, since all designated taxed groups will vote against the proposal, the coalition of designated taxed groups has to buy only as many untaxed groups as needed to form a simple majority against the proposal. Since the coalition of designated taxed groups has to buy fewer groups than the agenda setting group and because it is sufficient to offer these groups slightly more than the agenda setting group to win their vote, the coalition of designated taxed groups has a positive gain from vote buying and it will form the least expensive majority which will vote against the proposal. Hence, every proposal of the agenda setting group would be refused. Since the randomly selected agenda setting group will never receive positive subsidies, because of assumption TR 1, it will refuse to make a proposal. Therefore, the education of a society is impossible and the economy remains in the state of backwardness.

It could be assumed that the failure in the education of a society may also depend on the fact that the whole society is initially in the poverty trap. Suppose that some groups are already educated. However, this would only have an effect on the size of the expected tax burden because educated individuals could be taxed more heavily. This implies that the vote buying budget of the agenda setting group and the coalition of designated taxed groups respectively is larger if the coalition of designated taxed group is already educated. This, in turn, would only have an effect on the size of the payment
offers to the untaxed groups. Since the coalition of designated taxed groups will make its payment offers after the agenda setting group has announced its proposal, it will still be able to bid in such a way that the proposal of the agenda setting group will be rejected. Hence, an unequal income distribution in the society will not affect the impossibility result.

In the appendix we also show that the total number of designated taxed groups has no influence on the result derived above. Recall that the agenda setting group has to buy all untaxed groups in order to win the vote, whereas the coalition of designated taxed groups has to buy only as many untaxed groups as needed to form a simple majority that will vote against the proposal. Suppose, for example, that the agenda setting group will tax \( T \) groups, where \( 1 \leq T \leq (n - 1)/2 \). Note that, because of assumption TR 2, all designated taxed groups will vote against the proposal. In order to win the vote, the agenda setting group has to make vote buying as costly possible for the coalition of designated taxed groups. That is, the agenda setting group has to make equal payment offers to \( n - 1 - T \) untaxed groups, whereas the designated taxed group has to buy only \( (n + 1)/2 - T \) groups and offer these groups slightly more than the agenda setting group. Hence, the number of taxed groups is also irrelevant for the impossibility result.

In the following section we show how the negative effects of vote buying can be eliminated by introducing appropriate constitutional rules.

### 7.2 The Result under a Repeated Voting Constitution

Since we have assumed that monitoring of voters is not possible for the agenda setting group, the use of direct sanctions, e.g. the threat of punishment for vote buyers and sellers respectively, may be inadequate to prevent vote buying. Moreover, in reality it is difficult to find evidence for election fraud. Therefore, we have to develop other constitutional rules to eliminate the practice of vote buying.

A suitable constitutional principle is to allow the agenda setting group repeated voting on its proposal.\(^\text{20}\) The additional agenda rule could be described as follows:

\(^\text{20}\) The concept of repeated voting constitutions to remove opposition to socially desirable projects has been introduced by Gersbach (2006) for the provision of public goods. In our context, we will use the repeated voting constitution to break the blockade against human capital enhancing proposals induced by vote buying. Repeated voting may also have other virtues. For example, Morton (1988) has shown that agents can acquire information on voter preferences by observing the results of early referenda and use that information in formulating a strategy for subsequent referenda.
• **Repetition of Voting (RoV(R))**: If the proposal of an agenda setting group $i$ is rejected, the voting on that proposal can be repeated. A vote can be repeated $R$ times. If the proposal is accepted, repetition ends. However, if the proposal is rejected $R$ times, status quo continues to prevail.

We now describe the sequence in period $t$ in more detail: At the beginning of period $t$, the agenda setting group is allowed to make a proposal. In the following, we focus on proposals where the agenda setting group will tax $(n - 1)/2$ groups and only subsidize itself. In the next stage the society holds a vote on the implementation of this proposal. If a majority votes in favor of the proposal, then the proposal is accepted. Otherwise, there will be a revote on this proposal. If the project is rejected again, then there will be a further voting on this subject. This procedure can be repeated for as long as the proposal is not accepted. However, repetition of voting stops if the proposal is rejected $R$ times. In this case the status quo will prevail.

If buying of votes is possible, we obtain:

**Proposition 2**

Consider the case in which there is a democracy with $C\{BB, RoA, TFM(\tau^m, \tau), RoV(R)\}$, 

$$\pi = \min\{\pi_{sub}, \tau^*\},$$

where $\pi_{sub}$ is the highest taxation allowed for members of groups in a state of backwardness and $\tau^*$ is the highest tax burden for members of an already subsidized group that does not endanger schooling in the future, and the number of possible voting repetitions amounts to

$$R = [R^*]^{21} \quad \text{with} \quad R^* = \frac{n - 1}{2} \cdot \frac{\Delta\pi_{sub}}{\varepsilon}, \quad \varepsilon > 0.$$  

Such a democracy can educate a society in finite time, i.e., $T < \infty$, when vote buying is possible.

The formal proof is given in the appendix.

The result of proposition 2 is that repeated voting constitutions (RoV) in combination with a threshold flexible majority rule (TFM) may enable a society to escape from the poverty trap. The reason is the following: In the appendix we show that the possibility of repeated voting makes vote buying more costly because the coalition of designated taxed groups has to buy at least one untaxed group in each vote to form a minimal coalition against the proposal. If the number of possible voting repetitions is sufficiently large, then vote buying would not be profitable for the coalition of designated taxed

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21 Note that, since $R^*$ is a positive real number, we use the ceiling function to denote the minimal natural number larger than or equal to $R^*$.
groups, since total payments needed to prevent the implementation of the proposal will outweigh tax demand of the first round. Hence, it is optimal for the coalition of designated taxed groups not to engage in vote buying in the first round. Thus, the proposal of the agenda setting group will pass the legislation. The threshold flexible majority rule guarantees that households could not be adversely taxed. Because of the rotating agenda setting rule (RoA), each single interest group will be in office and receive the required transfer $\Delta \bar{s}$. Hence, the education achieved in the period of transfer is sustainable and consequently the society will be educated in finite time.\footnote{Gersbach and Siemens (2005) have developed further suitable rules for overcoming the problem of excessive taxation, for example, a taxpayer protection rule. Note that a repeated voting constitution in combination with a taxpayer protection rule may also make the education of a society possible.}

In order to limit redistribution and thus inequality in the society, we can introduce an additional restriction. This restriction could be described as follows: Only proposals of the agenda setting group are allowed to be repeated where the share of the tax revenue for the agenda setting group does not exceed a certain threshold. For example, the agenda setting group is only allowed to keep at most $\Delta \bar{s}/B_t$ of the tax revenues for itself. This restriction states that the agenda setting group can channel subsidies amounting to at most $\Delta \bar{s}$ to itself. Given this restriction, the best strategy of the agenda setting group is to divide the remaining tax revenues equally among the untaxed groups to make each untaxed group equally expensive for the coalition of designated taxed groups to invade. Note that we have assumed that the optimal choice of education, $e^0(\lambda_t)$, is increasing with human capital and income respectively. That is, if the subsidies for the untaxed groups are sufficiently large, it will have a positive effect on the educational choice of their members. Hence, not only the children of the agenda setting group would enjoy full-time schooling but also the children of the untaxed groups would have at least some (basic) school education. This, in turn, would imply that the possibility of vote buying and repeated voting constitutions in combination with a constraint on the subsidization of the agenda setting group may substantially reduce the time needed to educate the society.

However, it is clear that RoV will only work if repeated vote buying actually reduces the wealth of the vote purchaser. The success of a repeated voting constitution could possibly be in danger if "long-term vote buying contracts" are possible. A long-term vote buying contract is a binding agreement that gives the vote buyer full control of the vote of another group for more than one vote in exchange for an up-front payment. However, we believe that long-term vote buying contracts are not feasible in reality. The reason is that vote buying contracts are informal and therefore they cannot be
enforced by courts. Vote sellers, for example, may have strong incentives to renegotiate in future votings in order to enhance their welfare. Moreover, potential vote sellers may not vote as instructed if future proposals of the agenda setting group will make them better off. However, if long-term vote buying contracts are feasible, then further rules have to be developed.

7.3 The Result under a Flexible Minority Rule

An alternative instrument is a threshold flexible minority rule where the required share of votes is smaller than 1/2 if the proposal is constitutional. In our context, a constitutional proposal is accepted if a minority votes in favor of the project. Correspondingly to the threshold flexible majority rule, we define

\[ \tau_t^{\text{max}} = \max_{i \in \{1, \ldots, n\}} \tau_t^i. \]

- **Threshold flexible minority rule (TFMinority[\tau_t^{\text{max}}, \tau])**: Under this rule, the share of votes required to implement a proposal, \( v(\tau_t^{\text{max}}, \tau) \), jumps from \( \frac{2}{n} \) to 1 if any group \( i \) is taxed higher than the threshold tax \( \tau \) stated in the constitution:

\[
v_i(\tau_t^{\text{max}}, \tau) = \begin{cases} 
\frac{2}{n} & \text{if } \tau_t^{\text{max}} \leq \tau; \\
1 & \text{if } \tau_t^{\text{max}} > \tau.
\end{cases}
\]

In our context, as long as the proposal is constitutional, i.e., \( \tau_t^{\text{max}} \leq \tau \), it will pass the legislation if at least two groups vote in favor of the project. That is, for the rejection of the proposal, \( n - 1 \) groups have to vote against the proposal. However, unconstitutional proposals, i.e., \( \tau_t^{\text{max}} > \tau \), could be blocked by a single group.

It might be possible that the subsidies for the agenda setting group do not suffice for sustainable education, i.e. \( \Delta s_t^a < \Delta s \). In this case, repeated subsidization is needed to provide the children of the agenda setting group with permanent education. Therefore, additional to the rotating of the agenda setting rule, we define

- **Agenda repetition (AR)**: The agenda adopted in period \( t \) can be repeated \( K \) times in the subsequent periods if one-time subsidization is not sufficient to provide full-time education for the agenda setting group. Therefore, an agenda setting group is only chosen every \( K \) periods.

We summarize our findings in the following proposition:
Proposition 3
Consider the case in which a democracy with \( C\{BB, RoA, AR, TFMinority(\tau^{\text{sub}}, \pi)\} \), with \( \pi = \min\{\tau^{\text{sub}}, \tau^*\} \), where \( \tau^{\text{sub}} \) is the highest taxation allowed for members of groups in a state of backwardness and \( \tau^* \) is the highest tax burden for members of an already subsidized group that does not endanger schooling in the future. Such a democracy can educate a society in finite time, i.e., \( T < \infty \), when vote buying is possible.

The formal proof is given in the appendix.

The result of proposition 3 is that a threshold flexible minority rule in combination with the possibility of agenda repetition may enable a society to escape from the poverty trap. In contrast to the constitution described in proposition 1 where a simple majority was needed to reject a constitutional proposal, only a share of \( 2/n \) is now required for the implementation of the project. That is, under the former decision rule the coalition of designated taxed groups has to buy only as many untaxed groups as needed to form a simple majority against the proposal. However, under the threshold flexible minority rule the coalition of designated taxed groups is now forced to buy all untaxed groups to prevent the implementation of the project. Since the agenda setting group has to buy only one untaxed group, it is now in a position to make a payment offer to an untaxed group so that vote buying is never profitable for the coalition of designated taxed groups. That is, the total subsidies of the agenda setting group to this untaxed groups are so high so that the coalition of designated groups cannot afford the additional payments needed to purchase the votes of the remaining untaxed groups. Hence, all untaxed groups will vote in favor of the proposal and because of the threshold minority rule it would be accepted. The main drawback of this rule is that the agenda setting group’s utility of winning, i.e. the difference of tax revenues and the total payments of the agenda setting group to the untaxed groups, can be small. In this case, repeated subsidization of the agenda setting group is required (AR). Moreover, the threshold flexible minority rule guarantees that households could not be adversely taxed and because of the rotating agenda setting rule (RoA), each single interest group will be in office and be receiving. Hence, the education achieved in the period of transfer is sustainable and consequently the society will be educated in finite time.

Note that subsidization of the untaxed group will have a positive effect on the educational choice of their members. Hence, not only the children of the agenda setting group would benefit from the agenda repetition but also the children of the subsidized untaxed group. This effect may compensate for the drawback that the agenda setting
group may need some periods to educate its members.

8 Conclusions

Our paper has granted two insights. First, we have shown that if agents can buy and sell votes, then the education of a democratic society is impossible and hence society will remain in the poverty trap. The reason for this pessimistic result is that a coalition of designated taxed groups will have opportunities to counterattack if the society holds a vote on a proposal of the agenda setting group. This negative result may provide one possible explanation, amongst others, as to why societies in many developing countries are caught in poverty trap for a long time. Note that our result that vote buying has a negative effect on the economic development ties in with the findings of Docquier and Tarbalouti (2001).

Second, we have presented two appropriate constitutions, namely the repeated voting constitution and the constitution with a threshold flexible minority voting rule, to eliminate the negative effects of vote buying. Repeated voting constitutions, in combination with a threshold flexible majority rule, may make the education of a society possible because the possibility of repeated voting would make vote buying for the designated taxed groups more costly and therefore less attractive. A constitution with a threshold flexible minority rule in combination with the possibility of repeated subsidization may also educate a society because the flexible minority rule substantially raises the vote buying costs for the coalition of designated taxed groups. The reason is that, in contrast to the flexible majority rule, the coalition of designated taxed groups is now forced to buy more untaxed groups to prevent the implementation of the project. The design of both constitutions demonstrates that it is necessary to provide the agenda setting group with more power compared to the case without vote buying in order to overcome the poverty trap.

Our analysis is based on some crucial assumptions. For example, we assume that vote buyers can perfectly monitor the casting of votes of the purchased groups. Obviously, relaxing this assumption would make vote buying more costly and thus less attractive. The reason is that the coalition of designated groups may be forced to buy more groups to form a supermajority against the proposal in order to compensate the potential loss of votes of deviating groups. If monitoring is not possible, then buying of votes may not be attractive for the coalition of designated taxed groups because the untaxed groups may take the money and vote in favor of the proposal of the agenda setting
group. Further crucial assumptions are that agents can buy votes at no charge, there are no moral costs of vote buying and the risk of vote buyers and sellers being arrested and punished is zero. Apparently, relaxing these assumptions would also make vote buying less likely to occur and the education of a society more likely. Moreover, we have assumed throughout the paper that the payment offers of the designated taxed groups are made up-front. One might wonder what would happen if vote buyers can make post-voting payments. However, post-voting payments may create a commitment problem because vote buying contracts cannot be enforced by courts. This may possibly make the occurrence of vote buying less likely.

There are interesting possible extensions of our model. For example, we have assumed that vote buying is a binding agreement that gives a group full control of the vote of another group in exchange for a cash payment. In reality, however, there are different forms of vote buying. In some developing countries, for example, clientelism is prevailing. That is, some groups have control over valuable resources, for example access to arable land, fertilizers, water and so on and they are in a position to exploit their “market power” and demand compliance from their clients who wish to have a share of those goods. It would be an interesting point for future research to examine whether the constitutional rules developed in this paper may be appropriate to overcome poverty in such a setting.
A Proofs

Proof of Proposition 1: We now show that in the presence of vote buying, even with rules such as the threshold flexible majority rule (TFM), the balanced budget rule (BB) and the rotating agenda setting rule (RoA), democracies may fail to educate a society.

Firstly, note that we use subgame perfect equilibrium as the solution concept. Also note that, since we have a finite game with perfect information, subgame-perfect equilibria in pure strategies can be found by backward induction. Therefore, we start by considering stage 2 of the vote buying game and analyze the reaction of the coalition of designated taxed groups for an arbitrary proposal of the agenda setting group.

Note that the agenda setting group can tax at most \((n - 1)/2\) groups if it wants to win the vote. Let \(T\) denote the number of designated taxed groups, where \(0 < T \leq (n - 1)/2\). Hence, the number of designated untaxed groups amounts to \(n - T - 1\). Initially all groups are caught in the poverty trap, \(\lambda_i^0 = 1 \forall i \in \Omega\), i.e., the agenda setting group can levy taxes amounting to at most \(\Delta \tau^\text{sub}\) on each taxed group. Moreover, all groups know which group will be taxed and which will receive subsidies if the proposal is accepted. The expected total tax revenue of the agenda setting group is therefore given by:

\[
B_i = T \Delta \tau^\text{sub}
\]  

(10)

In the following, we show that any proposal which includes payment promises to untaxed groups will lead to zero subsidies for the agenda setter, which implies that the randomly chosen agenda setter will never apply for agenda setting. Consider a proposal of the agenda setting group which includes payment promises to untaxed groups in order to form a coalition of untaxed groups that supports the proposal. Since the coalition of designated taxed groups will move second and attack the weakest part of the agenda setting group’s coalition, the best strategy of the agenda setting group is to make each untaxed group equally expensive for the coalition of designated taxed groups to invade.\textsuperscript{23} That is, the agenda setting group will offer the same amount of subsidies to all \(n - T - 1\) untaxed groups. It is clear that the best the agenda setting

\textsuperscript{23}Subsequent to this proof, we show in lemma 1 that proposals with unequal payment offers or proposals with payment offers only to \((n - 1)/2\) groups to ensure a simple majority could also be prevented by vote buying. Since we have assumed that the coalition of designated taxed groups will buy the “cheapest” untaxed groups in order to form a simple majority against the proposal, such proposals would only reduce the total amount of payments which have to be paid by the coalition of designated taxed groups in order to form a minimal winning coalition.
group can do in order to win the vote is to use the entire expected tax revenues, $B_t$, to purchase votes. The highest possible payment offer of the agenda setting group to each designated untaxed group $i \in NT$ is therefore given by:

$$\Delta s_i^* = \frac{B_t}{n - T - 1} \forall i \in NT$$  \hspace{1cm} (11)

Inserting (10) in (11) yields:

$$\Delta s_i^* = \frac{T}{n - T - 1} \Delta \tau_{sub} \forall i \in NT$$  \hspace{1cm} (12)

After the announcement of the proposal, the coalition of designated taxed groups may have an incentive to buy votes of the untaxed groups in order to prevent the implementation of the proposal. Therefore, we now consider the payment offers of the coalition of designated taxed groups. Note that, since we have assumed that unbribed groups will vote against the proposal if $\tau_i^d > 0$, all taxed groups will vote against the proposal. That is, for the coalition of designated taxed groups it suffices to buy $(n + 1)/2 - T$ designated untaxed groups to form a minimal coalition that will vote against the proposal. The coalition of designated taxed groups has to pay each of them the amount of

$$\frac{T}{n - T - 1} \Delta \tau_{sub} + \varepsilon, \quad \varepsilon > 0$$  \hspace{1cm} (13)

in order to prevent the realization of the agenda setting group’s proposal.

However, the coalition of designated taxed groups will buy votes if, and only if, $B_t > \sum_{i \in NT} \Delta p_i^d$, holds, i.e., if the following condition is fulfilled:

$$\varepsilon < \frac{n - 3}{(n + 1 - 2T)(n - 1 - T)} T \Delta \tau_{sub} := \bar{\varepsilon}$$  \hspace{1cm} (14)

Recall our assumption that $n > 3$. Since $0 < T \leq \frac{n - 1}{2}$, $\bar{\varepsilon}$ is positive, i.e., the potential gain of vote buying for the coalition of designated taxed groups is positive and therefore it is profitable for the coalition of designated taxed groups to engage in vote buying. According to assumption VS, comparing (12) with (13) yields that the groups which are “purchased” by the coalition of designated taxed groups will vote against the proposal. Hence, the proposal is not adopted.

We now turn to the first stage of the vote buying game. Due to the tie-breaking rule TR 1, the agenda setting group will never apply for agenda setting because, for any

24 Note that the following argumentation also holds for proposals where $\Delta s_i^w > 0$.

25 Note that for $n = 3$, $\bar{\varepsilon}$ amounts to zero. That is, in this case the untaxed group receives payments amounting to $\Delta \tau_{sub}$ from the agenda setting group and therefore, because of assumption VS, it votes in favor of the proposal. The subsidies for the agenda setting group, however, will amount to zero.
possible proposal, it will never receive a positive subsidy. Hence, the human capital in
the next period amounts to \( \lambda_{t+1}=1 \) for all groups and the education of a society is
not possible.

\[ \square \]

**Lemma 1**
If the agenda setting group makes a proposal, then the best strategy of the agenda
setting group is to make each untaxed group equally expensive for the coalition of
designated taxed groups to invade, i.e., to play a leveling strategy.

**Proof:**
We now show that in the vote buying game the best strategy of the agenda setting
group is to play a leveling strategy. Therefore, we show that proposals with unequal
payment offers would only reduce the total amount of payments which have to be
paid by the coalition of designated taxed groups in order to form a minimal winning
coalition.

Firstly, let \( S_t \) denote the total amount of payments of the agenda setting group to the
untaxed groups in period \( t \):

\[
S_t := \sum_{i \in NT} \Delta s^i_t
\]

Secondly, assume that the condition VB(1)

\[
B_t > S_t
\]  

holds. In the following we will call the strategy where each of the \( n - \mathcal{T} - 1 \) untaxed
groups will receive the same amount, i.e., \( \Delta s^i_t = \frac{S_t}{\frac{n - \mathcal{T} - 1}{n - 1}} \forall i \in NT \), a leveling strategy.

We now turn to proposals with unequal payment offers. Let \( NT_1 \) denote the set where
\( \Delta s^i_t < \frac{S_t}{\frac{n - \mathcal{T} - 1}{n - 1}} \forall i \in NT_1 \) and \( NT_2 \) denote the set where \( \Delta s^i_t \geq \frac{S_t}{\frac{n - \mathcal{T} - 1}{n - 1}} \forall i \in NT_2 \). Moreover, let \( |NT_1| \) be the total number of groups belonging to set \( NT_1 \). Correspondingly, let \( |NT_2| \) be the total number of groups belonging to set \( NT_2 \). Note that \( NT_1 \cup NT_2 = NT \) and the total number of untaxed groups, \( |NT| \), amounts to \( n - 1 - \mathcal{T} \). Condition (15) can now be rewritten as:

\[
B_t > \sum_{i \in NT_1} \Delta s^i_t + \sum_{i \in NT_2} \Delta s^i_t
\]  

Recall that for the coalition of designated taxed groups it suffices to buy \( \frac{(n+1)}{2} \mathcal{T} \) groups to form a minimal coalition that will vote against the proposal and we assume
that the coalition of designated taxed groups will buy the cheapest groups. Note that
the coalition of designated taxed groups has to pay each of these groups the amount \( \Delta p_t^i = \Delta s_t^i + \varepsilon \) in order to win their vote. For \( \frac{n+1}{2} - T < |NT_i| \) it is obvious that the value of winning for the coalition of designated taxed groups i.e. the difference between the expected tax burden and the payments to untaxed groups needed to reject the proposal, \( B_t - \sum_{i \in NT_i} \Delta p_t^i \geq 0 \), is greater than in the case where the agenda setting group is playing a leveling strategy.

Now consider the case where \( \frac{n+1}{2} - T \geq |NT_i| \). Let \( |NT_i| \) and \( |NT_2| : = \frac{n+1}{2} - T - |NT_i| \) be the number of untaxed groups that the coalition of designated taxed groups needs to form a minimal winning coalition. \( |NT_2| \) is given by \( N - 1 - T - |NT_i| \). Note that \( |NT_2| - |NT_2'| \) groups with payment offers of the agenda setting group amounting to \( \Delta s_t^i \geq \frac{S_t}{n-1} \) are not purchased by the coalition of designated taxed groups. In contrast, under the leveling strategy the payment offers of the agenda setting group for the \( |NT_2| - |NT_2'| \) groups would amount to \( \Delta s_t^i = \frac{S_t}{n-1} \). Hence, the value of vote buying under the leveling strategy amounts to \( \frac{S_t}{n-1} \cdot (|NT_2| - |NT_2'|) \). In contrast, the value of vote buying for proposals with unequal payment offers arises from \( (|NT_2| - |NT_2'|) \cdot \Delta s_t^i \) which is weakly higher than under the leveling strategy, and from strictly lower payments to \( |NT_i| \) groups compared to the outcome with a leveling strategy.

This implies that the value of winning for the coalition of designated taxed groups is greater than in the case where the agenda setting group is playing a leveling strategy. Hence, we have shown that proposals with unequal payment offers would only reduce the total amount of payments which have to be paid by the coalition of designated taxed groups in order to form a minimal winning coalition.

\[ \square \]

**Proof of Proposition 2:**

We now show that under the constitution C with the threshold flexible majority rule (TFM), the balanced budget rule (BB), the rotating agenda setting rule (RoA) and the repetition of voting rule (RoV), democracies may be able to educate a society.

In the following, we are looking for subgame-perfect equilibria in pure strategies which can be found by backward induction. Therefore, we will derive the condition for the number of repetitions of voting, where vote buying is not profitable for the coalition of designated taxed groups. The repetitions of votes are indexed by \( r \in \{0, 1, \ldots, R\} \). Let \( B_t(r) \) denote the total expected tax revenue if the voting is repeated \( r \)-times. Let \( \Delta s_t^i(r) \) denote the subsidy that group \( i \in NT \) would receive from the agenda setting group, if the proposal is accepted after it was rejected \( r \) times before. Correspondingly, let \( \Delta p_t^i(r) \) denote the payments that group \( i \in NT \) would receive from the coalition.
of designated taxed groups, if it again votes against the proposal which was already rejected \( r \) times before.

At the beginning of period \( t \), the agenda setting group is allowed to make a proposal. In the following, we focus on proposals where the agenda setting group will tax \((n - 1)/2\) groups and pay no subsidies to the untaxed groups, i.e. \( \Delta s_i^j(r) = 0 \ \forall i \in NT \) and \( \forall r \in \{0, 1, \ldots, R\} \). Recall that initially all groups are caught in the poverty trap, \( \lambda^*_i = 1 \ \forall i \in \Omega \). That is, the expected tax revenues in the first round, i.e., when the number of repetitions is zero, are given by

\[
B_t(0) = \frac{n - 1}{2} \Delta \tau^{sub}
\]

(17)

Since the agenda setting group will tax \((n - 1)/2\) groups it suffices for the coalition of designated taxed groups to buy one untaxed groups to form a minimal coalition that will vote against the proposal and to pay this group

\[
\Delta p_t(r) = \varepsilon, \ \varepsilon > 0
\]

in each voting. In order to win \( R \)-times the voting against the proposal, the coalition of designated taxed groups has to pay the total amount of:

\[
\sum_{r=0}^{R} \Delta p_t(r) = [\Delta p_t(0) + \Delta p_t(1) + \ldots + \Delta p_t(R)] = R \cdot \varepsilon
\]

(18)

We are now able to derive the number of repetitions of this proposal which ensures that vote buying will not be attractive for the coalition of designated taxed groups. The condition where vote buying is never profitable for the coalition of designated taxed groups is given by:

\[
\sum_{r=0}^{R} \Delta p_t(r) \geq B_t(0)
\]

Hence, the number of repetitions \( R^* \) where vote buying will be not profitable for the coalition of designated taxed groups is implicitly given by:

\[
\sum_{r=0}^{R^*} \Delta p_t(r) = B_t(0)
\]

(19)

We now return to equation (19) in the light of equations (17) and (18). We obtain

\[
R^* \cdot \varepsilon = \frac{n - 1}{2} \Delta \tau^{sub}
\]

(20)
Rearranging equation (20) yields

$$R^* = \frac{n - 1}{2} \frac{\Delta \tau^{sub}}{\varepsilon}$$

Since $R^*$ is a positive real number, we have to use the ceiling function for $R^*$. The ceiling function is denoted by $\lceil R^* \rceil$ and it denotes the minimal natural number larger than or equal to $R^*$.

To sum up: If the voting is only repeated $R < \lceil R^* \rceil$ times, then it is profitable for the coalition of designated taxed groups to buy votes, since $\sum_{r=1}^{R} \Delta p_i(r) < B_i(0)$. However, if $R \geq \lceil R^* \rceil$, then vote buying is not profitable for the coalition of designated taxed groups, since VB(2) is not fulfilled. Thus, applying backwards induction, it is optimal for the coalition of designated taxed groups not to engage in vote buying. Hence, the proposal will be accepted in the first voting.

We now show that it is not profitable for the agenda setting group to deviate if voting on a proposal can be repeated $R^*$ times. First, it is not profitable for the agenda setting group to tax more than $(n-1)/2$ groups. This would imply that the proposal would never be accepted, since a majority would always vote against the proposal, i.e., the subsidies for the agenda setting group would amount to zero. Second, it is not profitable for the agenda setting group to tax fewer than $(n-1)/2$ groups. Obviously, these proposals would be accepted since the coalition of designated taxed groups now has to buy now more than one group to win each vote. However, taxing less than $(n-1)/2$ groups would entail a reduction of the subsidies for the agenda setting group. Hence, taxing fewer than $(n-1)/2$ groups is not profitable for the agenda setting group. Third, it is not profitable for the agenda setting group to pay positive subsidies to the untaxed groups. It appears that these kind of proposals would also be accepted, since subsidizing of untaxed groups would make vote buying more costly and therefore less attractive for the coalition of designated taxed groups. However, subsidizing of untaxed groups would also lead to lower subsidies for the agenda setting group. Hence, subsidizing of untaxed groups is also not profitable for the agenda setting group.

The remaining part of this proof is analogous to the proof of proposition 4 in Gersbach and Siemens (2005). Therefore, we only give a brief sketch. The threshold flexible majority rule, TFM($\tau_i^{max}, \tau$), with $\tau = \min\{\tau^{sub}, \tau^*\}$, where $\tau^{sub}$ is the highest taxation allowed for members of groups in a state of backwardness and $\tau^*$ is the highest tax burden for members of an already subsidized group that does
not endanger schooling in the future, is used to prevent adverse taxation. Hence, TFM(τ_i^{max}, τ) ensures sustainable, full-time education for the offspring of the agenda setting group. The corresponding flexible majority rule is given by:

\[ m_i(τ_i^{max}, τ) = \begin{cases} \frac{1}{2} & \text{if } τ_i^{max} ≤ τ; \\ 1 & \text{if } τ_i^{max} > τ; \end{cases} \]

where \( m_i(τ_i^{max}, τ) \) denotes the required majority depending on the maximum tax rate levied on the households. Because of the Rotating agenda setting rule (RoA), each single interest group will be in office and receive the required transfer \( Δτ \). Hence, the education achieved in the period of transfer is sustainable and the society will be educated in finite time.

\[ \square \]

**Proof of Proposition 3:**

We now show that under the constitution C with the threshold flexible minority rule (TFMinority), the balanced budget rule (BB) and the rotating agenda setting rule (RoA), democracies may be able to educate a society.

In the following, we focus on proposals where the agenda setting group will tax \( T \) groups, where \( 1 ≤ T ≤ n - 2 \), and subsidize itself and one designated untaxed group. Recall that initially all groups are caught in the poverty trap, \( λ_i^0 = 1 \ ∀i ∈ Ω \). That is, the expected tax revenues are given by

\[ B_i = TΔτ^{sub} \]  \hspace{1cm} (21)

Note that, in contrast to the constitution in proposition 1 where a simple majority was needed to reject a constitutional proposal only a share of \((n - 2)/n\) is needed to prevent the implementation of the project. That is, the coalition of designated taxed groups is now forced to buy all untaxed groups to prevent the implementation of the project whereas the agenda setting group needs the vote of only one untaxed group to win the voting. In order to prevent the proposal, the coalition of designated taxed groups has to pay the amount \( ε \) to each of \( n - 2 - T \) non-subsidized untaxed groups. That is, the total amount is given by \((n - 2 - T)ε\). Moreover, the coalition of designated taxed groups also has to buy the subsidized group. The payments to this group amount to \( Δs_i^1 + ε \). Hence, total payments to the untaxed groups which are needed to prevent the
proposal are given by:

$$
\sum_{i \in NT} \Delta p_i^i = (n - 2 - T)\varepsilon + (\Delta s_i^i + \varepsilon)
$$

$$
= (n - 1 - T)\varepsilon + \Delta s_i^i
$$

The condition where vote buying is never profitable for the coalition of designated taxed groups is given by:

$$
\sum_{i \in NT} \Delta p_i^i \geq B_t
$$

We are now able to derive the amount of subsidies the agenda setting group has to pay to the untaxed group which guarantees that vote buying is not profitable for the coalition of designated taxed groups. It is given by:

$$
\Delta s_i^i = B_t - (n - 1 - T)\varepsilon
$$

(22)

Inserting (21) into (22) yields:

$$
\Delta s_i^i = T\Delta \tau^{su} - (n - 1 - T)\varepsilon
$$

(23)

We now turn to the subsidies for the agenda setting group. Recall that the subsidies for the agenda setting group are given by:

$$
\Delta s_i^{as} = B_t - \Delta s_i^i
$$

(24)

Inserting (21) and (23) into (24) yields:

$$
\Delta s_i^{as} = (n - 1 - T)\varepsilon
$$

Obviously, the agenda setting group will only tax one group, since subsidies for the agenda setting are maximal if $T = 1$. According to assumption VS, the subsidized untaxed group will vote in favor of the proposal. Moreover, because of TR 2, all non-subsidized untaxed groups will also vote in favor of the proposal. Hence, the proposal will be adopted.

Note that, if $\Delta s_i^{as} = (n - 2)\varepsilon \geq \Delta \pi$, then the education of the children of the agenda setting is sustainable. However, if $(n - 2)\varepsilon < \Delta \pi$, then the education of the children of the agenda setting is not sustainable. That is, in the case of a one-time subsidization, the lineages of this group will fall back into the poverty trap over time. Consequently repeated subsidization is required. Hence, in this case we need a further constitutional rule, namely the agenda repetition rule (AR). Recall that under the agenda repetition
rule the agenda setting group is allowed to repeat its proposal $K$ times in the subsequent periods if one-time subsidization is not sufficient to provide sustainable education for the agenda setting group. We use $K^*$ to denote the minimal number of periods a continuously subsidized household of the agenda setting group needs to accumulate human capital of at least $\lambda^* + \epsilon$. Suppose, that $s_t^{a_s} < \bar{s}$. Then, $K^*$ is implicitly given by:

$$\lambda_{t+K^*} = \sum_{k=0}^{K^*} \left\{ z h [\epsilon (\alpha + s^{a_s})] \right\}^k = \lambda^* + \epsilon$$

To sum up: The possibility of agenda setting repetition ensures that the member of the agenda setting group will be fully educated. The threshold flexible majority rule, $\text{TFV}(\tau_t^{\text{max}}, \bar{\tau})$, with $\bar{\tau} = \min\{\tau_{\text{sub}}, \tau^*\}$, is used to prevent adverse taxation. Hence, $\text{TFV}(\tau_t^{\text{max}}, \bar{\tau})$ ensures sustainable education for the offspring of the agenda setting group. Because of the rotating agenda setting rule (RoA) each single interest group will be in office and receive the required transfers for full-time schooling. Hence, the education achieved in the period(s) of transfer is sustainable and the society will be educated in finite time.

$\square$
References


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