Borda rule is intended also for dishonest men

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Abstract

This paper studies the welfare consequences of strategic voting in Borda rule by comparing the utilitarian efficiencies in simulated voting games under two behavioural assumptions: expected utility maximising behaviour and sincere behaviour. Utilitarian efficiency is higher with expected utility maximising behaviour than with sincere voting behaviour. Strategic voting increases utilitarian efficiency particularly if the distribution of preference intensities correlates with voter types and if the voters have at least some aggregate level information on preference intensities. Borda rule is shown to have two advantages related to strategic voting. Strategic voting is beneficial in this rule even if some but not all voter types engage in strategic behaviour and strategic voting is beneficial even if the voters' information is based on unreliable signals if those signals contain some intensity information.

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1 Introduction

One of the main criticisms of Borda rule is that it is highly susceptible to strategic voting (see e.g. Saari 1990*b*, Favardin et al. 2002, Smith 1999). Borda is famous for having exclaimed "My scheme is intended only for honest men" (quoted in Mueller 1989, p. 112) when the susceptibility of his rule to strategic manipulation was pointed out.

This paper studies the welfare consequences of strategic voting in Borda rule using computer simulations. As in Lehtinen (2004, 2005), the welfare consequences of strategic voting are evaluated by comparing the *utilitarian efficiency* obtained with *Expected Utility maximising voting behaviour* (EU-behaviour) and with *Sincere Voting behaviour* (SV-behaviour). In SV-behaviour *all* voters always vote sincerely. In EU-behaviour, the voters may vote strategically or sincerely depending on their incentives. *Utilitarian efficiency* is defined as the percentage of simulated voting games in which the candidate with the highest sum of utility (the utilitarian winner) is selected. We will say that strategic voting is *welfare-increasing* if EU-behaviour generates higher utilitarian efficiencies than SV-behaviour and *welfare-diminishing* if the converse holds.

The main result is that strategic voting increases utilitarian efficiency as compared to sincere voting behaviour when the voters engage in expected utility maximizing behaviour under incomplete information. Under a utilitarian evaluation of the voting outcomes, what has been thought to be a major disadvantage of Borda rule turns out to be an argument *for* it. However, since strategic voting increases utilitarian efficiency in most commonly used voting rules (Lehtinen 2004, 2005), our results do not provide an unambiguous argument for using Borda rule instead of some other voting rule. On the other hand, and in contradistinction to majority rule in amendment agendas, we will see that Borda rule yields high utilitarian efficiencies even when the voters' information on the other voters' preferences is fairly unreliable, and even if some but not all voters engage in strategic behaviour.

The proponents of Borda rule have traditionally argued that it selects fair compromises as outcomes. Indeed, Borda himself seems to have argued for his rule by referring to cardinal utilities. He argued that with three candidates, and in the absence of further knowledge, the preference for the second-place candidate could be assumed to be midway between the best and the worst (de Borda 1995[1784], p. 85). We will see that strategic voting yields welfare diminishing results precisely when the utilities for the middle candidates are on the average midway between the worst and the best. To put this more exacly, strategic voting is welfare-diminishing or less welfare-increasing when the utilities for the middle candidates are uniformly distributed in the interval $[0, 1]$ between the worst and best candidates, but welfare-increasing when the second-place utilities for *some* candidates are typically higher than the average of the uniform distribution (i.e. higher than one-half), and the second-place utilities for some *other* candidates are typically lower than this average. Computer simulations that use such assumptions will be called *setups with intensity correlation* because the preference intensities (utilities for the middle candidate) correlate with voter types; voters with some preference orderings have typically a high or low utility for their middle candidate.

Strategic voting increases utilitarian efficiency mainly because it allows the voters to express intensities of preference. The utilitarian winner is most likely to obtain strategic votes. To understand why, consider the *counter-balancing* of strategic votes; Under incomplete information, some voters may have an incentive to decrease the Borda score for candidate x and increase that of y

while at the same time some other voters may have an incentive to increase the Borda score of candidate y and decrease that of candidate x. Voting strategically *for* (*against*) a candidate means giving it a higher (lower) Borda score than the voter's preference ordering would imply.[1] Since voting strategically for a candidate is more likely when the utility for that candidate is high than when it is low, a utilitarian winner is likely to obtain *more* strategic votes *for* and *less* strategic votes *against* it than other candidates. Strategic voting thus provides fuller information on intensities of preference than sincere voting.[2] This suggests that the Borda rule itself may not need to be made probabilistic (Heckelman 2003) or fuzzy (García-Lapresta & Martínez-Panero 2002, Marchant 2000) in order to obtain information on intensities.

The title of this paper is neither a joke nor a metaphor. It should be taken literally. In many other voting rules, strategic voting may be considerably less welfare-increasing or welfare-diminishing if some but not all voter types engage in strategic behaviour (see Lehtinen 2004, 2005). We will argue that the beneficial welfare consequences of strategic voting in Borda rule do not depend crucially on the assumption that *all* voter types engage in strategic behaviour. Unlike other voting rules, Borda rule is fairly robust to this kind of heterogeneity in behavioural dispositions.

The structure of the paper is the following. Section 2 presents our assumptions on the voters' preferences. Section 3 formulates an expected utility model of strategic voting in Borda rule. Section 4 introduces a model of incomplete information; signal extraction in simulated games which is explained in more detail in Lehtinen (forthcoming). Section 6 presents simulation results. Section 7 concludes.

2 Voter types and preferences

Let X={x,y,z} denote the set of candidates (with generic members j and k).[3] The six possible types of voters and their preference orderings are presented in table 1 below. Let U_1, U_2, and U_3 denote a voter's utility for her best, second-best, and worst candidate, respectively.

v_i denotes the voter's *intensity of preference*, $0 < v_i < 1$. The voters assign their best candidate utility one, and the worst candidate utility zero. Let $\mathfrak{N}_j^1$ be the *number* of voters who put candidate $j \in X$ highest in their preference ordering. Let $\mathfrak{N}_j^2$ denote the *set* of voters who consider candidate j as second-

[1] Strategically manipulating Borda rule by introducing a new alternative or manipulation by coalitions is not considered in this paper. See Dummett (1998) and Saari (1990*a*) for the former and Lepelley & Mbih (1994) and Lepelley & Valognes (2003) for the latter.

[2] Donald Saari (e.g. 2003) has consistently argued in favor of using Borda rule instead of majority rule. His argument has been that the latter throws away information but the former does not.

[3] The present model is restricted to three candidates only. The framework of this paper (the signal extraction information model) can easily be extended to more than three candidates. All that is needed is an account of expected utility maximization with more than three candidates.

type of voter						
t_1	t_2	t_3	t_4	t_5	t_6	U_i
x	y	z	x	y	z	$U_1 = 1$
y	z	x	z	x	y	$U_2 = v_i$
z	x	y	y	z	x	$U_3 = 0$

Table 1: Voter types and utilities

best. The sum of utility, $U(j)$, for any candidate $j \in X$ is:

$$U(j) = \mathfrak{N}_j^1 + \sum_{i \in \mathfrak{N}_j^2} v_i \tag{1}$$

The utilitarian winner, UW, is the candidate with the largest sum of utility:

$$UW = max\{U(x), U(y), U(z)\}. \tag{2}$$

Our criterion of welfare incorporate interpersonal comparisons of preference intensities. Since such comparisons are generally regarded as unacceptable for epistemic and conceptual reasons (interpersonal comparisons are meaningless with vNM utility functions), we need to justify our welfare criterion.

We rely on interpersonal preference intensity comparisons for the following reasons.[4] The only way of explicitly evaluating the welfare consequences of strategic voting in a *welfarist* manner is by comparing the *voting outcomes* under different behavioural assumptions (EU-behaviour and SV-behaviour) with *identical preferences* and institutional assumptions. But voting outcomes cannot be evaluated without making interpersonal comparisons. A Condorcet winner is an alternative that has a majority against all other alternatives. The Condorcet is an alternative to the utilitarian winner for making normative evaluations on voting outcomes. The notion of a Condorcet winner also incorporates an interpersonal comparison, because any two alternatives X and Y are compared by the number of individuals who would gain utility in passing from X to Y with the number who would lose (cf. Hildreth 1953). Therefore, *any* explicit investigation of the welfare consequences of strategic voting will have to make *some* interpersonal comparisons.

The main alternative concept for evaluating strategic voting in simulated voting games, Condorcet efficiency, is not based on a *more* respectable epistemological argument than the utilitarian winner because the very possibility of strategic voting implies that we cannot obtain reliable information on individual preferences. The *normative* judgment that each voter has an equal 'weight' (in that each voter's utility ranges from 0 to 1) in the social welfare function is compelling in many voting contexts.

The main argument for using utilitarian instead of Condorcet efficiency, however, is that it allows for a welfarist comparison of the consequences of strategic

[4] See Hammond (forthcoming, 1991) for surveys on interpersonal comparisons, and Black (1976) for some discussion on interpersonal comparability with respect to Borda rule in particular.

voting in different voting rules without being biased in favor of some voting rules. Finally, the simulations can be run with various different interpersonal comparisons to see if they are robust in this respect.

3 A model of strategic voting in Borda rule

Borda rule is defined as follows.[5] Let n denote the number of candidates. The voters are asked to provide a full ranking of all candidates. A voter assigns n-1 marks for the top alternative, n-2 for the second candidate,.., 0 for the worst candidate. The Borda winner is defined as the candidate that obtains the largest sum of marks, i.e. the largest *Borda count*.

Let 1, 2, and 3 denote the best, second-best, and the worst candidate. Let $p(12)$ denote the probability that the best candidate obtains a higher Borda count than the second-best candidate. $p(13)$ and $p(23)$ are similarly defined. Reporting the ordering 123 then means voting sincerely, and reporting any other order means voting strategically.

There are two possible motivations for voting strategically in Borda rule.[6]

Situation 1 The voter believes that her best candidate might win the contest, and she does not like the second-best candidate very much. In order to secure the victory of the best candidate, she gives the lowest score to her second-best candidate. She must simultaneously believe that her strategic vote is not likely to make the worst candidate win. Intuitively, the voter compares the probability that her putting candidate 3 second in the ranking brings victory for 3, with the probability that 1 wins the whole contest if she votes strategically by lowering 2's Borda score.

- This situation is characterized by the following kinds of probabilities and preferences: p(13) high, p(12) high, p(23) high, and U_2 low.
- When a voter votes with this motivation, she reports 132 instead of 123.

Situation 2 The voter believes that her best candidate does not have a chance of winning, but her second-best candidate might have a chance, and she likes this second-best candidate somewhat intensively. In order to increase the chance that this second-best candidate wins, she puts it first, the best candidate second, and the worst candidate last.

- This situation is characterized by the following kinds of probabilities and preferences: p(12) low, p(13) low, p(23) high, and U_2 high
- When a voter votes with this motivation, she reports 213 instead of 123.

[5] See Pattanaik (2002) for a review of the axiomatic literature on Borda rule and other positional methods.

[6] To the best of my knowledge, there are no incomplete information models of strategic voting in Borda rule. Black (1976) and Ludwin (1978) provide an account that resembles the first situation, and Felsenthal (1996) considers a case that resembles the second.

We may now formulate a general decision rule for our voters: They vote sincerely unless they are in situation 1 or 2:

Situation 1. Vote strategically by reporting 132 if

$$p\,(12)\,p\,(13)\,p\,(23) - U_2 > \tau_1, \tag{3}$$

where τ_1 is a threshold. The threshold reflects the voters' propensity to engage in strategic voting.

Situation 2. Vote strategically by reporting 213 if

$$p\,(23)\,U_2 - p\,(13)\,p\,(12) > \tau_2, \tag{4}$$

where τ_2 is a threshold[7].

The conditions for strategic voting for all voter types are presented in tables 2 and 3.

voter type	Situation 1	vote as though type
type 1: xyz	$p(xy)p(xz)p(yz) - U(y) > \tau_1$	4: xzy
type 2: yzx	$p\,(yz)\,(1 - p(xy))(1 - p(xz)) - U(z) > \tau_1$	5: yxz
type 3: zxy	$[1 - p(xz)]\,[1 - p(yz)]\,p(xy) - U(x) > \tau_1$	6: zyx
type 4: xzy	$p(xz)p(xy)\,[1 - p(yz)] - U(z) > \tau_1$	1: xyz
type 5: yxz	$[1 - p(xy)]\,p(yz)p(xz) - U(x) > \tau_1$	2: yzx
type 6: zyx	$[1 - p(yz)]\,[1 - p(xz)]\,[1 - p(xy)] - U(y) > \tau_1$	3: zxy

Table 2: Conditions for strategic voting in the first situation

voter type	Situation 2	vote as though type
type 1: xyz	$p(yz)U(y) - p(xz)p(xy) > \tau_2$	5: yxz
type 2: yzx	$[1 - p(xz)]\,U(z) - [1 - p(xy)]\,p(xy) > \tau_2$	6: zyx
type 3: zxy	$p(xy)U(x) - [1 - p(yz)]\,[1 - p(xz)] > \tau_2$	4: xzy
type 4: xzy	$[1 - p(yz)]\,U(z) - p(xy)pxz > \tau_2$	3: zxy
type 5: yxz	$p(xz)U(x) - p(yz)\,[1 - p(xy)] > \tau_2$	1: xyz
type 6: zyx	$[1 - p(xy)]\,U\,(y) - [1 - p(xz)]\,[1 - p(yz)] > \tau_2$	2: yzx

Table 3: Conditions for strategic voting in the second situation

Consider now counter-balancing of strategic votes. Consider first situation 1. Let us assume, as in simulations with intensity correlation, that the intensities for candidate x ($U\,(x)$ in the table above) are on average higher than those for candidate y, ($U\,(y)$ in the table above). These assumptions imply that x is likely to be the utilitarian winner, and y the worst outcome in utilitarian terms. If voters of type 1 or 6 vote strategically in situation 1, their strategic vote *decreases* the Borda score of y, and increase that of x and z, respectively. In

[7] It is possible to give parameters τ_1 and τ_2 different values, but in this paper they were assumed to be the same in all except those simulations in which one of these parameters was so large (i.e. at least 1) that there is no strategic voting in that situation.

contrast, if voters of type 3 or 5 vote strategically in situation 1, their strategic vote *decreases* the Borda score of x, and increase that of y and z, respectively. The conditions in table 2 imply that, under our assumptions, voters of type 1 and 6 are likely to vote strategically *more often* than voters of type 3 and 5.

A high average intensity for x implies that in situation 2, *many* type 3 and 5 voters will vote strategically by raising the Borda score of x at the expense of y and z. A low average intensity for y implies that *few* type 1 and 6 voters vote strategically. They are thus likely to refrain from making y's Borda score higher.

Hence, relatively *many* strategic votes *for* the utilitarian winner x are likely to be counter-balanced by relatively *few* strategic votes *against* it. We will return to discussing counter-balancing in section 6.1.

4 The voters' signals and beliefs

4.1 Model-consistency

The basic idea of our information model is that the voters formulate beliefs on the basis of noisy signals of the other voters' preferences.[8] This information model is embedded in simulated games that are based on the *impartial culture* (Tsetlin, Regenwetter & Grofman 2003, Gehrlein 2002) assumption.

A *simulated game g* consists of a set of payoffs lotted by a random number generator, beliefs based on these payoffs and other informational assumptions, and voting outcomes under the different behavioural assumptions. The uniform distribution on [1,2,...,6] (i.e. the impartial culture) was used in all simulations considered in this paper to generate a profile of voters $\{1, 2, ..., N\}$ in each simulated game g.[9]

It has been argued that strategic voting is fairly difficult in Borda rule, because voting strategically requires knowledge of the full preference orderings of all the other voters. In fact, since the voters need to take into account other voters' strategic behaviour, one might even argue that it is necessary to know *each* voters' beliefs. Our model of incomplete information does not require knowledge on individual-level utilities or beliefs. Evaluating which candidates have a chance of winning the vote is all that is important for the voters. This kind of information can be obtained from perturbed signals concerning pairwise aggregate-level comparisons between the candidates.

It is certainly not plausible to assume that the voters know every other voter's preference intensity. However, they may well have some aggregate-level information on these intensities. In order to incorporate game-theoretical considerations into our model in a *model-consistent* (Muth 1961) manner, we need

[8] Lehtinen (forthcoming) discusses the signal extraction model in more detail.

[9] We used 201 voters in the simulations. This particular number for chosen mainly to obtain comparability with some earlier simulation studies of voting rules (Merrill 1984, 1988). It is easy to study the effect of the number of voters on the results but this was not done for the present paper.

to assume that the voters take information on preference intensities into account. To see why, consider what the voters may know about other voters' strategic voting.

If the voters maximise expected utility, the difference between the voting results from EU-behaviour and SV-behavior is that the former allows preference intensities to affect the results but the latter does not. Strategic voting increases the chance that candidates with a relatively large sum of utility are selected. Hence, if the voters engage in EU-behaviour, a candidate's sum of utility predicts her chance winning the vote reasonably well. Therefore, if and when the voters are able to take other voters' strategic behaviour into account, they will do so by assuming that candidates with high average utility will obtain more votes than information on mere preference orderings would imply. We will thus have to assume that the voters have some, albeit imprecise, information on the aggregate intensities for the various candidates.

Although there is no single unambiguous source of such information in the real world, it is plausible to assume that the voters have some indirect knowledge on aggregate-level intensities. The support for some candidates, for example, may be geographically limited, with fierce opposition elsewhere. The fact that a candidate's views may be radical or modest also provides some clues.

Since we will be interested in studying how the degree to which the voters' signals contain intensity information affects utilitarian efficiencies, we will assume that the voters obtain a convex combination of two kinds of signals; *preference signals* and *intensity signals*. Let $p^p(j,k)$ denote a probability concerning candidates j and k which is based merely on preference order information, and $p^u(j,k)$ a probability based on intensity information. The voters may thus formulate their probabilities based on an intensity signal $S^u(j,k)$ or a preference signal $S^p(j,k)$ or a combination of both. Let $\lambda \in (0.1)$ denote the *relative share of preference signals*. The voters are assumed to derive a probability based on the weighted sum of the probabilities derived from the two kinds of signals:

$$p(j,k) = \lambda p^p(j,k) + (1-\lambda)p^u(j,k). \tag{5}$$

The simulations were conducted with $\lambda = 1, 0.75, ..., 0$. When $\lambda = 1$, the probability is based only on preference signals and when $\lambda = 0$ the voters' probabilities are based on intensity signals.

4.2 Preference signals

When the voters obtain a preference signal, they obtain a perturbed signal of the *number* of voters who prefer a candidate to another. Let N denote the number of voters and $n(j \succ k)$ the number of voters who prefer candidate j to candidate k in simulated game g. Let $n_i(j \succ k) = 1$, if voter i prefers j to k, and $n_i(j \succ k) = 0$, if voter i prefers k to j. Then $n(j \succ k)$ can be viewed as a sum of N Bernoulli trials. The total number of voters who prefer j to k is thus $n(j \succ k) = \sum_{i=1}^{N} n_i(j \succ k)$. Let p denote the probability that such a Bernoulli trial results in the outcome $n_i(j \succ k) = 1$. The impartial culture implies that $p = \frac{1}{2}$.

$n(j \succ k)$ can thus be viewed as a random variable with a binary distribution $n(j \succ k) \sim B(N, \frac{1}{2})$. Since the number of voters who prefer j to to k can be viewed as a sum of N random variables, the Central Limit Theorem implies that it can be approximated with a normally distributed random variable $Q^p(j,k)$[10]. It will be more convenient to use such a standardized sum of Bernoulli trials instead of the variable $n(j \succ k)$. $Q^p(j,k)$ is given by

$$Q^p(j,k) = \frac{n(j \succ k) - Np}{\sqrt{Np^2}}. \tag{6}$$

Since $p = \frac{1}{2}$, this is $Q^p(j,k) = \frac{2n(j \succ k) - N}{\sqrt{N}}$. A *preference signal* of voter i concerning the preferences of all voters for alternatives j and k , $S_i(j,k)$, is given by

$$S_i^p(j,k) = \frac{2n(j \succ k) - N}{\sqrt{N}} + \varepsilon \cdot r_i(j,k), \tag{7}$$

where $r_i(j,k)$ is a realization of an i.i.d. standard normal random variable, and ε is a scaling factor that reflects the *reliability* of the signals. Let $R_i(j,k) = \varepsilon \cdot r_i(j,k)$. The signal can then be written as follows:

$$S_i(j,k) = Q^p(j,k) + R_i(j,k). \tag{8}$$

4.3 Intensity signals

Intensity signals are based on imprecise information on the sums of utilities for the candidates. The voters thus obtain perturbed signals concerning the difference in the *sum of utility* between *each pair* of candidates.[11]

Let $\Delta_i^u(j,k) = U_i(j) - U_i(k)$, and $\Delta^u(j,k) = \sum_{i=1}^N \Delta_i^u(j,k) = U(j) - U(k)$. A signal consists of the difference in the sum of utility $\Delta^u(j,k) = U(j) - U(k)$ and a random term $\varepsilon r_i(j,k)$. An intensity signal, $S_i^u(j,k)$, thus has the following form:

$$S_i^u(j,k) = U(j) - U(k) + \varepsilon r_i(j,k), \tag{9}$$

where $r_i(j,k)$ is a realization of an i.i.d. standard normal random variable, and ε again reflects the reliability of the signals.

By the Central Limit Theorem, the sum $\sum_{i=1}^N \Delta_i^u(j,k)$ of N random variables $\Delta_i^u(j,k)$ can be approximated with a normally distributed random variable $Q^u(j,k)$. The standardized variable $Q^u(j,k)$ is defined as follows:

$$Q^u(j,k) = \frac{\Delta^u(j,k) - E[\Delta^u(j,k)]}{\sigma_\Delta} = \frac{\Delta^u(j,k)}{\sigma_\Delta} = \frac{U(j) - U(k)}{\sigma_\Delta}, \tag{10}$$

[10] This approximatimation restricts the applicability of our model to situations with a fairly large number of voters.

[11] Notice that although we assume that the voters obtain perturbed signals of sums of utilities, this assumption does not entail any interpersonal comparisons. In fact, it could not, because interpersonal comparisons cannot be given by individual choices (Myerson 1985) but our voters use sums of utilities merely as a proxy for predicting other voters' choices.

where σ_Δ is the standard deviation of the variable $\Delta^u(j,k)$. Let $R_i(j,k) = \varepsilon \cdot r_i(j,k)$. An *intensity signal* of voter i is given by:

$$S_i^u(j,k) = Q^u(j,k) + R_i(j,k). \tag{11}$$

It can be shown that the standard deviation of Δ^u is $\sigma_\Delta = \frac{\sqrt{5N}}{3}$. The signal is thus given by

$$S_i^u(j,k) = \frac{3\Delta^u(j,k)}{\sqrt{5N}} + R_i(j,k). \tag{12}$$

The signals can be expressed as a sum of two normally distributed random variables $Q = \frac{2n(j \succ k) - N}{\sqrt{N}} \sim N(0, \sigma_Q^2)$ (or $Q = \frac{3\Delta^u(j,k)}{\sqrt{5N}} \sim N(0, \sigma_Q^2)$) and $R = R_i(j,k) \sim N(0, \sigma_R^2)$:

$$S = Q + R. \tag{13}$$

Calculating the probability that candidate j obtains a higher Borda count than candidate k, $p_i(j,k)$, given a signal $s_i(j,k)$, involves deriving a distribution function for the variable $X = (Q|S = s)$, and evaluating the probability that Q is smaller (or greater) than zero. Lehtinen (forthcoming) shows that

$$F(Q < 0|S = s) = \Phi\left(-\frac{\sigma_Q}{\sigma_R}\frac{1}{\sqrt{\sigma_Q^2 + \sigma_R^2}}s_i(j,k)\right) \tag{14}$$

Since $\sigma_Q^2 = 1$ and $\sigma_Q^2 = \varepsilon^2$, probabilities based on preference signals are given by

$$p_i^p(j,k) = 1 - \Phi\left(-\frac{s_i^p(j,k)}{\varepsilon\sqrt{1+\varepsilon^2}}\right), \tag{15}$$

and probabilities based on intensity signals are given by

$$p_i^u(j,k) = 1 - \Phi\left(-\frac{s_i^u(j,k)}{\varepsilon\sqrt{1+\varepsilon^2}}\right), \tag{16}$$

5 Setups and simulation

A setup is a combination of assumptions used in a set of $G = 10000$ simulated games. The EU-setups differ with respect to the degree of reliability of the voters' information (ε), the voters' propensity to vote strategically (τ), the relative share of intensity-information (λ), and the degree of correlation between voter types and preference intensities (C) (see next paragraph).

In *uniform setups* each voter's preference intensity (v_i) is drawn from a uniform distribution on [0,1]. In *setups with intensity correlation* some voter types have systematically more intense preferences for their second-best candidates than some other voter types. These setups are identical to the corresponding uniform setups with respect to all parameters except the voters' intensities. The intensities for candidates x and y are systematically different. The intensities for candidate x (v_i^3, and v_i^5) were drawn from a uniform distribution on $[C, 1]$, and the intensities for y (v_i^1, and v_i^6) from $[0, 1-C]$. v_i^2 and v_i^4 were drawn from the uniform distribution on $[0, 1]$. The average intensity for x, $\frac{1}{n_3+n_5}\sum_{i=1}^{n_3+n_5}(v_i^3 + v_i^5)$, is higher than the average intensity for y, $\frac{1}{n_1+n_6}\sum_{i=1}^{n_1+n_6}(v_i^1 + v_i^6)$. In the uniform setups the *degree of correlation* C is zero.

Let UW^g denote the utilitarian winner in a simulated game g, and $W_{EU}^g(\varepsilon, \tau, \lambda, C)$ the winner in an EU-behaviour setup. *Utilitarian Efficiency in an EU-setup* is given by:

$$UE_{EU}(\varepsilon, \tau, \lambda, C) = \frac{\sum_{g=1}^{G}(UW^g = W_{EU}^g(\varepsilon, \tau, \lambda, C))}{G} * 100. \quad (17)$$

Utilitarian Efficiency in the SV-setup, UE_{SV}, is calculated similarly. We will say that EU-behaviour is *welfare-increasing* in a setup if the utilitarian efficiency is higher in this EU-setup than in the SV-setup:

$$UE_{EU}(\varepsilon, \tau, \lambda, C) > UE_{SV} \quad (18)$$

If the converse holds, EU-behaviour is *welfare-diminishing*. We will also say that *strategic voting is welfare-increasing* in a setup if EU-behaviour is welfare-increasing in that setup.

6 Simulation results

We will be interested in studying how the degree of correlation (C), the reliability of the voters' information (ε), the voters' propensity to engage in strategic voting (τ), and the relative share of intensity-information (λ) affect utilitarian efficiency. All these parameters affect the results but we will have to keep at least two parameters fixed each time the results are reported.[12] The simulations were conducted with $C = 0, 0.05, ..., 0.5$, $\varepsilon = 0, 0.4, ..., 1.6$, $\tau = 0, 0.25, ..., 1$, and $\lambda = 1, 0.8, ..., 0$. Choosing the values for the parameters is to some extent arbitrary, but some choices can be motivated. For example, equation 15 implies that since the signal extraction model cannot be used with $\varepsilon = 0$ because it would involve dividing by zero, we never chose to present the results with this value of ε. Neither do we present results for $\tau = 1$ because the conditions for strategic voting imply that nobody votes strategically with $\tau = 1$[13].

Figure 1 displays utilitarian efficiencies in setups with different degrees of correlation. These results were derived with mere preference signals ($\lambda = 1$), and with $\varepsilon = 0.4$. It is easy to see from this figure that strategic voting is

[12] The FORTRAN codes to generate the results, and result tables with all possible combinations of parameter values can be obtained from the author on request. In order to check the computer code with the number of runs used (10000) for each setup, the IMSL library of FORTRAN codes and access to supercomputer are required. The simulations were conducted with a Sun Fire 25K server (UltraSPARC IV processor) at the Center for Scientific Computing, Otaniemi, Espoo, Finland.

[13] Simulations were conducted with $\tau = 1$, however, in order to check the consistency of the model. The results from SV-behaviour and EU-behaviour with $\tau = 1$ have to be the same.

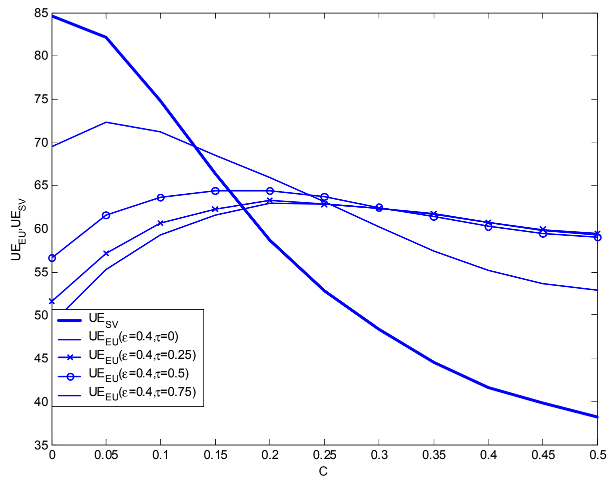

Figure 1: Utilitarian efficiencies in setups with different degrees of correlation. Voters obtained only preference signals ($\lambda = 1$) and the degree of reliability is not excessively low ($\varepsilon = 0.4$).

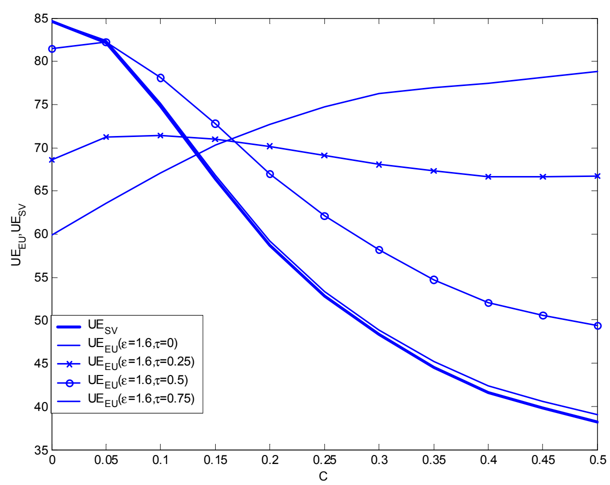

Figure 2: Utilitarian efficiencies in various setups with $\varepsilon = 1.6$.

welfare-diminishing if the degree of correlation between voter types and preference intensities (C) is less than about 0.17, and welfare-increasing if it is higher. The average intensity for x in this setup is $(1+0.17)/2 = 0.585$, and the average intensity for y is 0.415.

Figure 2 displays utilitarian efficiencies in setups with $\varepsilon = 1.6$. Comparing figures 1 and 2 shows that strategic voting is more welfare-increasing when the reliability of the voters' information is low than when it is high. The effect of the reliability of the voters' signals (the quality of the voters' information) on utilitarian efficiencies thus turned out to be surprising. As before, strategic voting is more welfare-increasing in setups with intensity correlation than in uniform setups.

These two figures also show that utilitarian efficiency is usually high when the voters' propensity to engage in strategic voting is high; setups with a low value of τ usually yielded higher utilitarian efficiencies than setups with a higher value. Furthermore, the more correlation there is, the more a high propensity of engaging in strategic voting increases welfare.

What happens if the voters obtain perturbed intensity information? Figure 3 displays the results in setups with $\varepsilon = 0.4$ when the voters have intensity information ($\lambda = 0$). These simulations used parameter values that were identical to those presented in figure 1 except that λ has changed from 1 to 0. As expected, if the voters have intensity information, even if this information is unreliable, strategic voting is very strongly welfare-increasing. Figure 4 shows that if the voters have intensity information, utilitarian efficiency is higher when this information is based on reliable rather than unreliable signals. Hence, whether having reliable signals is welfare-increasing as compared to having non-reliable signals depends on whether the signals contain intensity information or not. All in all it seems evident from an inspection of these figures that it is much more important what the voters' information is about than whether this information is highly reliable or not.

6.1 What if some voter types do not engage in strategic voting?

The logic of counter-balancing implies in many voting rules that if some voter *types* never vote strategically, strategic voting may be harmful (see Lehtinen 2005, Lehtinen 2004). If the strategic votes for a candidate are not counter-balanced with strategic desertions of the same candidate, the voting results no longer adequately reflect the differences in preference intensities between the candidates. It is unlikely, however, that the welfare consequences of strategic voting are affected at all if we select those who engage in SV-behaviour randomly, i.e. if there are no systematic differences between the different types of voters in their behavioural disposition. If we assume, for example, that a randomly selected half of the population of voters engages in SV-behaviour, there will simply be less strategic voting, and the model yields the same welfare effects as the model where all voters engage in EU-behaviour, but these effects are weaker.

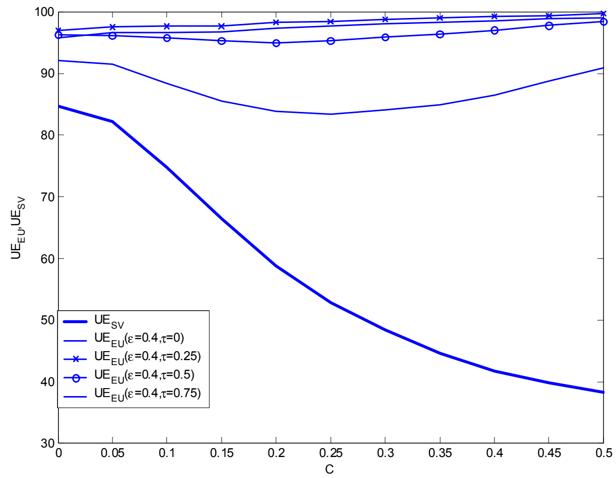

Figure 3: Utilitarian efficiencies with $\varepsilon = 0.4$ with intensity information ($\lambda = 0$).

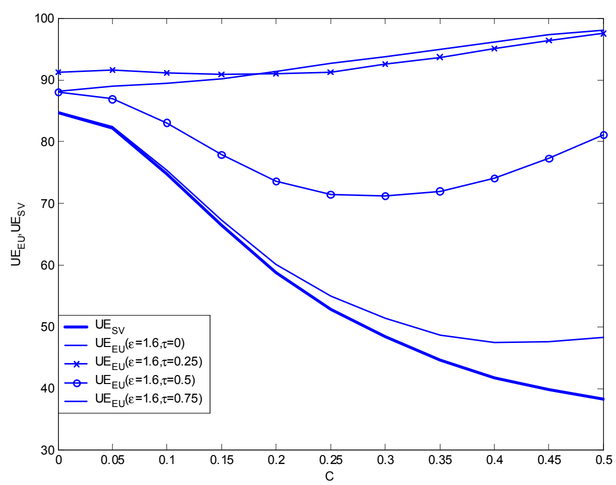

Figure 4: Utilitarian efficiencies with $\varepsilon = 1.6$ and $\lambda = 0$.

The main advantage of Borda rule to other voting rules is that the beneficial welfare consequences of strategic voting do not depend heavily on whether all voter types engage in strategic behaviour or not. The reason for this is that a single voter may confront two different strategic situations, and these situations have a different incentive structure. Although a voter *may* have an incentive of voting strategically for x in situation 1, she *may* also have an incentive of voting strategically against x in situation 2. The conditions of strategic voting presented in tables 2 and 3 imply, however, that a single voter cannot have an incentive to vote strategically in both situations at the same time. The possibility of two different strategic situations implies that strategic votes by one type of players need not be counter-balanced by strategic votes by *another type*(s) of voters because the logic of counter-balancing works at the level of a single individual.

It is not irrelevant, however, which voter type(s) do not engage in EU-behaviour. If voters of type 1 vote strategically, they do so by increasing the Borda score for z at the expense of y in situation 1, and by increasing the score of y at the expense of x in situation 2. If voters of type 2 vote strategically, they do so by increasing the score of x at the expense of z in situation 1, and by increasing the score of z at the expense of y in situation 2. Hence, type 1 voters never lower the score of z and type 2 voters never lower the score of x. A similar argument shows that type 3 voters never lower the score of y in voting strategically. It follows that utilitarian efficiencies should be higher in setups with correlation where only voters of type 1 (or 3) engage in SV-behaviour than in setups where only type 2 voters engage in SV-behaviour. The difference should be rather small, however, if only one type of voters refrains from strategic voting, because in these setups plenty of voters remain who may have incentives for voting strategically for all three candidates.

Figure 5 displays utilitarian efficiencies in setups where voters of type 1 engaged in SV-behaviour and other voter types engaged in EU-behaviour, and figure 6 displays similar results when type 2 voters engaged in SV-behaviour.

Comparing these figures to figure 1 shows that utilitarian efficiency is not affected in a dramatic way even though all voters do not engage in strategic behaviour. Furthermore, the difference between the setups in which voters of type 1 and of type 2 engaged in sincere behaviour is very small.

6.2 What if only one type of strategic situation is allowed?

We postulated in section 3 two different situations in which the voters may vote strategically. It turned out that there is a clear difference in the welfare implications between the two situations. Figure 7 displays utilitarian efficiencies from setups in which the voters engaged in strategic voting in situation 1 but not in situation 2, and figure 8 displays similar results for setups in which the voters engaged in strategic behaviour only in situation 2. These figures show that the welfare consequences were strongly beneficial if the voters engaged in strategic behaviour only in order to increase the victory chances of their most preferred candidate. If, in contrast, they only voted strategically to increase the chance that their second best candidate wins, the welfare consequences were much more mixed.

7 Conclusions

Strategic voting is welfare-increasing in Borda rule in various different configurations of assumptions. All our results derive from the *logic of counter-balancing*; intensively supported alternatives are most likely to gain strategic votes and least likely to lose them. Ceteris paribus, correlation between voter types and preference intensities makes strategic voting more welfare-increasing. If the voters have perturbed aggregate level intensity information on the other voters' preferences, utilitarian efficiency is usually high, even if this information is based on highly unreliable signals. Another way to put this is that if the voters are able to take other voters' strategic behaviour into account in a model-consistent manner, the welfare consequences of strategic voting are more beneficial than if they cannot take such strategic uncertainty into account.

It seems fairly likely that the setups with intensity correlation correspond more closely to real-world conditions than the uniform setups. This would be the case if some candidates were typically fairly tolerable to a large amount of voters even when they have about the same amount of supporters that put them first in their preference ordering (and some other candidates would have typically a narrower basis of supporters). Our results thus provide a further dimension to the claim made by various authors that Borda rule selects reasonable compromises; the utilitarian winner is one kind of a compromise candidate.

Although we have not conducted an explicit comparison of different voting rules in this paper, we can confidently claim that Borda rule has two advantages as compared to some other rules. First, strategic voting seems to be welfare-increasing even if the voters have unreliable information on other voters' preferences. Second, and more importantly, the welfare consequences of strategic voting are beneficial in this rule even if there is heterogeneity in the behavioural dispositions of the different types of players; even if some voter types do not engage in strategic behaviour, strategic voting increases utilitarian efficiency. The main reason for this is that counter-balancing functions even at the level of a single individual because there are two different strategic situations in Borda rule. We may thus conclude that Borda rule ought to have been intended for dishonest people also.

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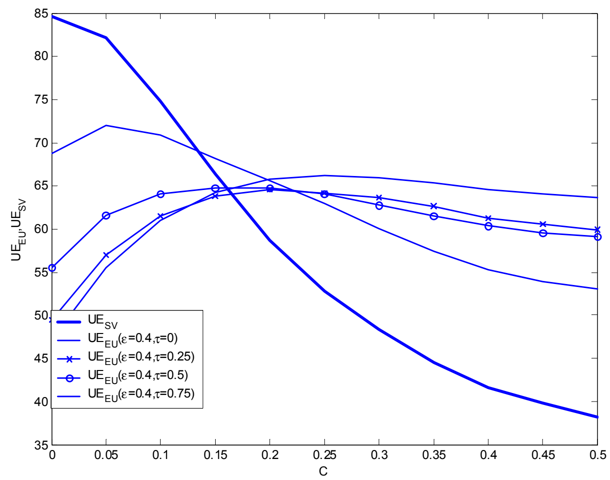

Figure 5: Utilitarian efficiencies with $\varepsilon = 0.4$ in setups with different degrees of correlation when voters of type 1 engage in SV-behaviour

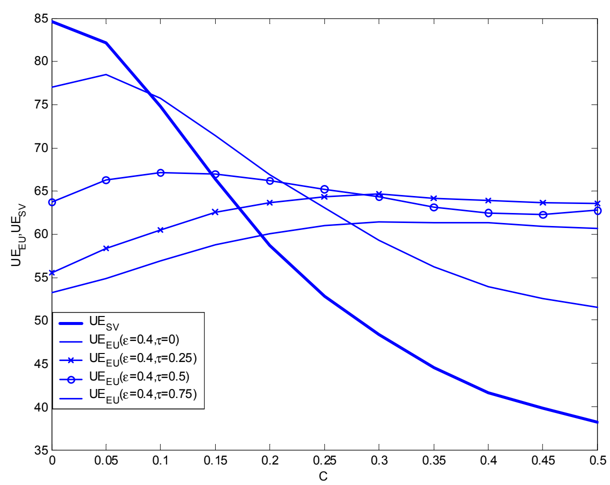

Figure 6: Utilitarian efficiencies with $\varepsilon = 0.4$ in setups with different degrees of correlation when voters of type 2 engage in SV-behaviour

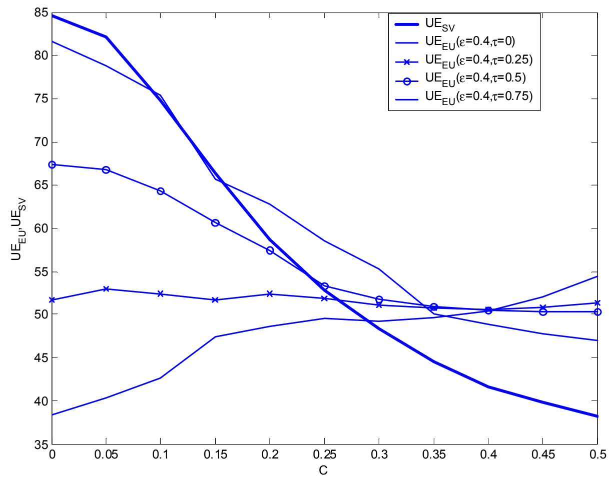

Figure 7: Utilitarian efficiencies in a setup where the voters engage in strategic behaviour only in situation 1 ($\varepsilon = 0.4$).

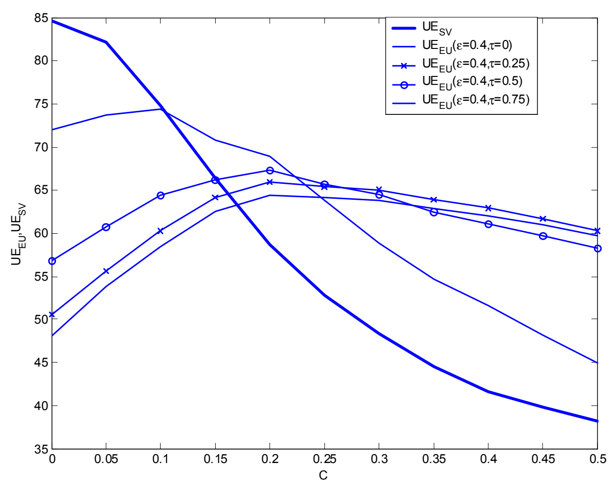

Figure 8: Utilitarian efficiencies in a setup where the voters engage in strategic behaviour only in situation 2 ($\varepsilon = 0.4$).